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Percolation in real wildfires

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Abstract. – This paper focuses on the statistical properties of wild-land fires and, in particular, investigates if spread dynamics relates to simple invasion model. The fractal dimension and lacunarity of three fire scars classified from satellite imagery are analysed. Results indicate that the burned clusters behave similarly to percolation clusters on boundaries and look denser in their core. We show that Dynamical Percolation reproduces this behaviour and can help to describe the fire evolution. By mapping fire dynamics onto the percolation models, the strategies for fire control might be improved.

In recent times the introduction of satellite imaging facilitated the coarse-scale analysis of wildfires [1]. Being inspired by the self-similar aspect of the fire scars, we want here to provide an explanation for this lack of characteristic length scale. We want to link wildfires spreading with the evolution of diffusive systems whose scale invariance has been widely analyzed [2]. In particular, we focus here on the simplest example of fractal growth model, the model of percolation [3]. Percolation has been extensively studied, and it proved to be extremely successful in explaining some of the statistical properties of several propagation phenomena ranging from the polymer gelation [4] to superconductors [5].

In this paper we present some evidence that percolation models could be fruitfully applied to describe properties of wildfires [6]. In particular, the dynamical version of percolation, known as Dynamical Percolation (DyP), may provide effective insights into the fire control. We based our analysis on the comparison between several statistical properties of wildfires with those of the percolation clusters. We report here the measures of the fractal dimension of areas, *accessible* perimeters and hulls (defined as the set of the most external sites of the

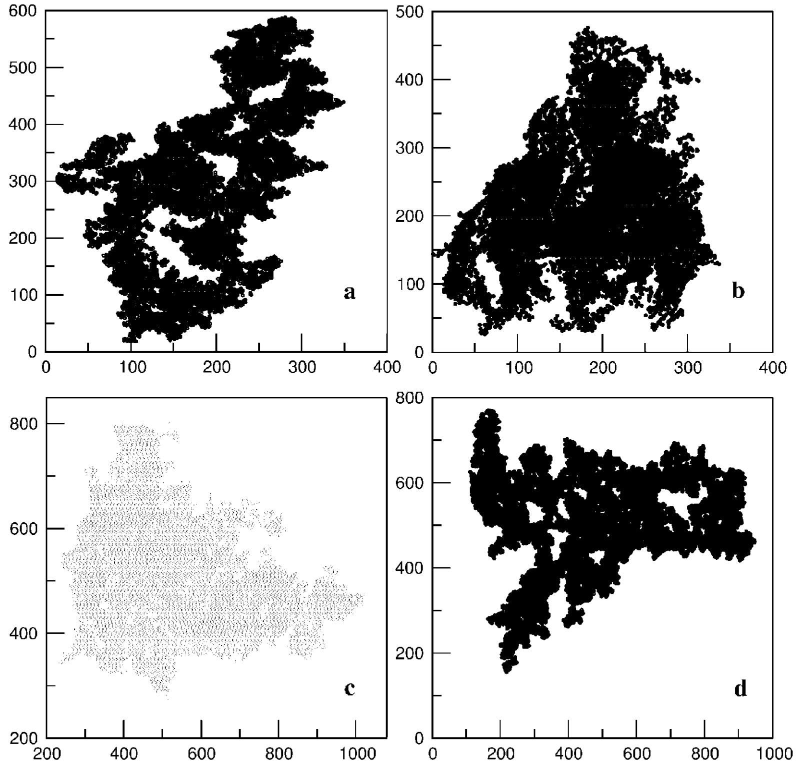

Fig. 1 – Binary map of the burnt areas a) for the valley of Biferno, b) for the Penteli wildfire, c) for the Cuenca wildfire. In d) we plot a cluster of Self-Organised Dynamical Percolation whose dimensions are comparable with case c). Each pixel corresponds to an area of $900\,\mathrm{m}^2$.

cluster) [3], along with the lacunarity (*i.e.* the void distribution inside the cluster). As a result, we can conclude that, within the error bars, statistical properties of wildfires can be accurately described by a self-organized version of DyP [7].

The data set shown here consists of Landsat TM satellite imagery ($30\,\mathrm{m} \times 30\,\mathrm{m}$ ground resolution) of wildfires, acquired, respectively, over the Biferno valley (Italy) in August 1988; over the Serranía Baja de Cuenca (Spain) in July 1994; and over the mount Penteli (Greece) in July 1995. In all the cases the image was acquired a few days after the fire. The burnt surfaces were, respectively, 58, 60, 156 square kilometers. Bands TM3 (red), TM4 (near infrared) and TM5 (mid infrared) of the post-fire subscene are classified using an unsupervised algorithm and 8 *classes* [8]. This means that in the above three bands any pixel of the image is characterised by a value related to the luminosity of that area. By clusterising in *classes* those values, one can describe different types of soil, and, in particular, the absence or presence of vegetation. In particular, the maps of post-fire areas have been transformed into binary maps where black corresponds to burned areas. These maps are shown in fig. 1.

In order to quantify the possible scale invariance, we measure the following properties: 1) the fractal dimension of the burned area; 2) the fractal dimension of the *accessible* perimeter; 3) the fractal dimension of the hull (defined as the set of burned sites on the boundary of the system); 4) the variance of the relative point density fluctuations (*i.e.* a measure of the lacunarity of the system). We compare these values with the corresponding ones of self-organized Dynamical Percolation that we believe to represent the phenomenon.

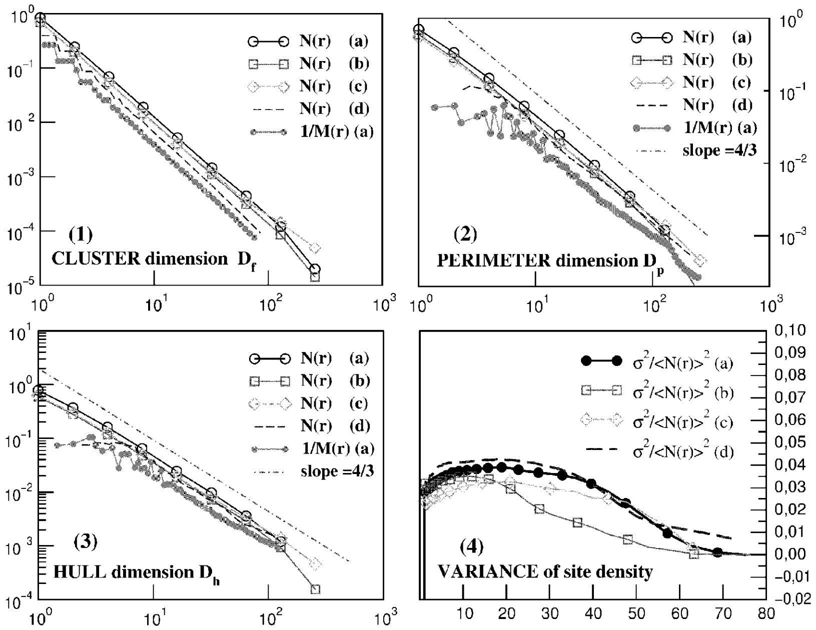

Fig. 2 – Scatter plot of 1) the fractal dimension of the aggregate, 2) the fractal dimension of the perimeter of the structure, 3) the fractal dimension of the hull of the structure, 4) the variance in the relative site density fluctuations. In all the plots we distinguish between cases a),...,d). For the fractal dimensions we report for case a) both the measures $N(r)$ and $1/M(r)$.

To measure fractal dimensions, we apply two different methods: the average mass-length relation and the box counting method. Box counting method is performed by overlapping a grid of size r over the data set and counting the number $N(r)$ of boxes occupied by the cluster at the scale r. For fractal objects $N(r) \propto r^{-D_\mathrm{f}}$ (for $r \to 0$), where D_f is the fractal dimension. The average mass-length relation measures the relationship between the average number $M(r)$ of points of the data set at distance r around any other point of the data set itself. This can be achieved by measuring $M(r)$ in a circle of radius r centered around a point of the system. For scale-free objects, $M(r) \propto r^{D_\mathrm{f}}$ (for $r \to \infty$). To avoid any bias in the result, circles should be fully included in the cluster. The two methods gave equal results within the error bars. In general, this means that D_f is a well-defined property of the system. These results for the box counting are shown in fig. 2 and summarized in table I. This table also reports the exact values that characterize critical percolation clusters. All wildfire measures but the hull are in very good agreement with percolation data.

TABLE I – *Fractal dimensions, for the data and for the DyP model. Exact values for DyP are computed on hierarchical lattices.*

	a) Biferno	b) Penteli	c) Cuenca	d) DyP
D_f	1.90(5)	1.93(5)	1.95(5)	91/48
D_h	1.30(5)	1.32(5)	1.31(4)	7/4
D_p	1.30(5)	1.33(4)	1.34(5)	4/3
$\mathcal{L}(1)$	0.037(3)	0.036(3)	0.034(3)	0.040(2)

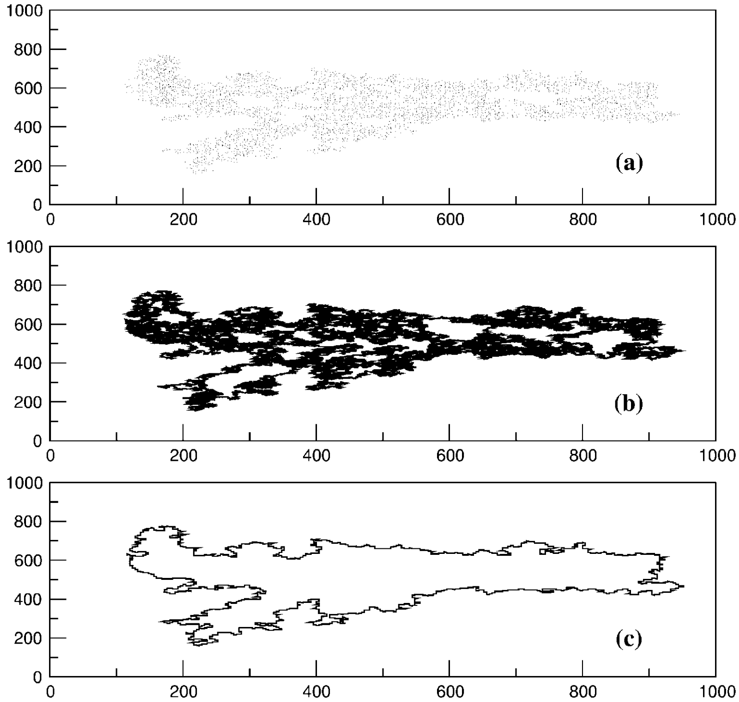

Fig. 3 – Numerical Simulation of a DyP cluster. In a) we show the cluster, in b) the hull of this cluster, in c) we show the hull of the coarse-grained picture of cluster in plot a). The coarse-grained version is obtained by using cells with linear size twice as the original one. After only two steps the statistical properties of the hull become similar to those of the perimeter.

We believe that this peculiar behaviour for the hull may depend on the coarse resolution of the remotely sensed burnt area (30 m), resulting in a kind of Grossman-Aharony effect [9] which reduces the hull of the critical percolation cluster to the accessible perimeter. Consequently, the hull fractal dimension (equal to 7/4) is reduced to the fractal dimension of the accessible perimeter (equal to 4/3). This effect can be induced by a different redefinition of the connectivity criterium on the hull sites. In this case in particular we refer to the studies of refs. [10,11], where a smaller than expected hull exponent is related to the low resolution of image with respect to the characteristic distance of the percolation process (see also ref. [12]). To test this assumption, a critical percolation cluster computer simulation with 3×10^5 sites was undertaken. Results show that, whereas most statistical properties do not change, the hull tends to behave as the accessible perimeter if a coarse-graining procedure is applied in such a way to reduce the resolution between first and second neighbors (see fig. 3).

The last measure we perform is the computation of the variance $\sigma(r)$ of the normalized point density fluctuations of the burnt sites, *i.e.* $\sigma(r) = \sqrt{\langle M(r)^2 \rangle / \langle M(r) \rangle^2 - 1}$. Generally, $\sigma(r)$ is a function of the radius r; for a "simple" fractal, the variance $\sigma(r)$ is a constant $\sigma_{\mathrm{intrinsic}}$. At large scale r, for values near to the spatial extension of the data set, the measure of statistical quantities ($M(r), \sigma(r)$, etc.) is affected by finite-size effects. Such effects produce a decrease of $\sigma(r)$ for increasing values of r. As we can see in fig. 2d), the measures of $\sigma(r)$ *vs.* r both for the fire data and the computer-simulated percolation cluster are in good agreement. Moreover, they fit the values reported in ref. [13], where the same quantity is estimated for an ordinary percolation cluster. The value $\sigma^2_{\text{intrinsic}}$ can be considered as a measure of the morphology of a fractal data set. The larger $\sigma^2_{\text{intrinsic}}$, the larger the probability that the fractal set has large voids. This is evident from fig. 2, where the variance has lower values for the the wildfires with smaller voids (sets b) and c)).

As a last remark on the data interpretation, we checked that the fuel load distribution before fires was rather uniform in the analyzed areas. Therefore, we can exclude that fractal properties of fire depend on pre-fire vegetation distribution. Nevertheless, additional work is underway to quantify the effects of pre-fire fuel load distribution on fire behaviour.

From the above data analysis it seems that percolating clusters could describe reasonably well the process of fire spreading. Unfortunately, in the original formulation percolation is a static model where one considers sites on a lattice that can be selected with a certain probability p. If $p = 1$ all points are selected and there are plenty of spanning paths in the system. When $p = 0$ no point is selected and there is no way to form a spanning path. By increasing p step by step from zero, small clusters of connected areas are generated, until for a particular value of probability $p = p_c$, called *percolation threshold*, a part of the small clusters coalesce and form a spanning cluster.

Even if most of the properties measured in real fires are reproduced by percolation, we need a model whose dynamics could mimic in a reasonable way that of the wildfires. We then propose here to use the Self-Organised version of Dynamical Percolation. Dynamical percolation [14] was introduced to study the propagation of epidemics in a population and its definition is the following: each site of a square lattice can be in one of three possible states: i) ignited sites, ii) green sites susceptible to be ignited in the future, and iii) immune sites (*i.e.* burned sites non-susceptible to be ignited again). At time $t = 0$ a localized seed of ignited sites is located at the center of an otherwise empty (green) lattice. The dynamics proceeds in discrete steps either by parallel or by sequential updating as follows: at each time-step every ignited site can ignite a (green) randomly chosen neighbor with probability p or, alternatively, burn completely and become immune to re-ignition with complementary probability $1 - p$. Any system state with no burning site is an *absorbing configuration*, *i.e.*, a configuration in which the system is trapped and from which it cannot escape [15, 16]. It is clear that depending on the value of p the fire generated by the initial ignited seed will either spread in the lattice (for large values of p) or die out (for small values of p). The two previous phases are divided at the percolation threshold p_c, where the fire propagates marginally, leaving behind a fractal cluster of immunized sites. Interestingly, it can be shown, using field-theoretical tools that this is a critical percolation cluster [14]. In this way we have a dynamical model which, at criticality, reproduces the (static) properties of standard percolation. Clearly, this model presents extensive fractal properties only if $p = p_c$. The tuning of this parameter exactly to p_c is, however, quite unlikely.

For that reason we present here then a *self-organized* version of this model assuming a time-dependent form for the ignition probability $p(t)$ decreasing from an initial value $p_0 > p_c$ with time constant τ (*e.g.*, $p_0 \exp[-t/\tau]$ or $p_0/[1+(t/\tau)^n]$). In the optics of fires, p_0 represents the initial "force" of the fire, and τ is its characteristic duration. This observation came from the experience in fire control. Even without human activity fires eventually stop. It is then fair to introduce a fire extinction probability increasing with time. Fire will then invade new regions and will be able to continue until the percolation probability is larger than or equal to the critical value p_c. This peculiar process is also able to reproduce in a qualitative way the results of the fire clusters. Indeed the fire will grow almost in a compact way at the beginning, leaving a fractal boundary at the end of the activity.

Let us suppose to start the dynamics at $p_0 > p_c$. At the beginning the dynamics is the same of DyP with constant $p > p_c$, *i.e.* the ignited region is quite compact, leaving only small holes of vegetation. However, with the time passing $p(t)$ reduces and then the diameter of islands in the burning cluster increases. Finally, after a certain time proportional to τ one has $p(t) < p_c$, then the dynamics arrests *spontaneously* in some time-steps. In particular, one can see that if $\tau \gg 1$, at the arrest time t_f, $p(t_f) \simeq p_c$. Therefore, the geometrical features of the final burnt cluster become more and more irregular (fractal) going towards the hull, representing an effective spatial (radial) probability of ignition. One can show that the final hull and the accessible perimeter have the same fractal dimensions for ordinary percolation. However, this fractality is extended only up to a characteristic scale $\xi \sim \tau^{\alpha_\xi}$, with $\alpha_\xi = 1/D_h$. ξ gives also the characteristic scale of the voids nearby the hull, which are the largest in the cluster. Moreover, $p_c - p(t_f) \sim \tau^{-\alpha_p}$, where $\alpha_p = (D_h - 1)/D_h$. In a word, the hull presents the main features found in another static percolation model known as Gradient Percolation [17]. We believe that these properties of the self-organized DyP explain why in the largest analyzed wildfires (*i.e.* starting with a larger p_0 or a larger τ) we have an effective increase of the global fractal dimension D_f towards 2 and the appearance of large voids only nearby the hull. Instead, for the smallest ones one can think that p_0 is too near p_c (with respect to the value of τ) to realize the spatial gradient of ignition probability. This would result in fire clusters as large as ξ, and then with the same features of ordinary clusters of percolation near criticality.

The importance of such a result lies in the particular growth dynamics shown by DyP. As pointed out in refs. [7, 15], DyP grows mainly by selecting sites newly added to the system. If this applies also to the external boundaries of a wildfire, one could in principle think to focus the activities of fire control where the fire invasion is faster. Here it is also worth discussing the features of the so-called "forest fire model" (see [18] for a complete review on the model). In this model at successive time steps trees (sites) are removed trough simple rules of ignition from nearest neighbours or by burning through external lighting. At the same time new trees grow on the empty sites left by the fire. This model not only presents the unrealistic assumptions of fast re-growing trees in the system, but also produces almost compact clusters of wildfires that fail to reproduce the statistical properties we observe in the data. We believe that despite the name, this model fails to reproduce the behaviour of real wildfires representing instead a nice statistical model to show the properties of Self-Organised systems.

In conclusion, we present here some of the statistical properties of wild-land fires. Results indicate that the cluster formed by fire shows, at least on boundaries, well-defined fractal properties strictly related to percolation at criticality. In particular, we believe that the most suitable model to describe fire dynamics is a self-organized version of Dynamical Percolation. Unfortunately, due to the coarse spatial and temporal resolution of available satellites, it is very difficult to check the dynamical properties of the forest fires. Nevertheless, the very good agreement between DyP and real data and the similar evolution of growth suggests that DyP could indeed represent a suitable model in most cases. The assignment of random probability values to the links between sites, which is performed in the model construction, can effectively model the broad-scale cumulative effect of interacting features (terrain, vegetation, etc.). Since the dynamical properties of DyP have been extensively studied, recognition of DyP dynamics for fire spread has important consequences for fire control. This should focus on the latest zones attacked by fire, since the zones left behind have a small probability to keep the fire alive. It is indeed a well-known result that DyP grows through the most recent areas entered in the system.

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