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The flaming sandpile: self-organized criticality and wildfires

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Abstract

A large series of wildfire records of the Regional Forest Service of Liguria (northern Italy) from 1986 to 1993 was examined for agreement with power-law behavior between frequency of occurrence and size of the burned area. The statistical analysis shows that the idea of self-organized criticality (SOC) applies well to explain wildfire occurrence on a regional basis.

Keywords: Fractal statistics; Liguria; Self-organized criticality; Wildfires

1. Introduction

Bak et al. (1988) proposed the notion of self-organized criticality (SOC) to describe the behavior of extended dynamical systems which persistently operate at states of critical equilibrium with no length or time scales others than those deduced from the size of the system and the elementary activation mechanism. This idea was intended to explain the abundance of fractals in nature (Mandelbrot, 1982), i.e. the presence of scale-free structures that look 'alike' on many scales of observation. Every attempt to split up fractal structures into smaller parts results in the resolution of the same pattern, no matter at which scale the pattern is examined.

Scale-invariance manifests itself algebraically through power-laws. A measure M of a set F with measurement unit size δ, is related to δ by the expression:

$$M_\delta (F) \propto \delta^{-D} \tag{1}$$

where D is the fractal (i.e. from the Latin 'fractus', meaning broken or non-integer) dimension reflecting the scale-invariant spatial clustering of F (Johnson et al. 1995). SOC has therefore been suggested as a possible explanation for the power-law (fractal) behavior seen in many natural systems, such as the frequency and magnitude of earthquakes (Bak and Tang, 1989; Sornette and Sornette, 1989; Ito and Matsuzaki, 1990; Ito, 1995), avalanches (Noever, 1993), and acoustic emissions from volcanic rocks (Diodati et al., 1991).

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The classical SOC experiment which simulates the statistics of this behavior is illustrated by a simple pile of sand built by randomly adding a grain at a time onto a flat, circular platform (Bak et al., 1988). The sandpile will grow increasing its slope until at any position on the pile the slope becomes too steep and few grains slide down causing a microscopic avalanche. If we simply continue to add grains, the slope of the pile steepens and the average size of the avalanches becomes larger. Some grains begin to fall off the edge of the platform. The pile stops growing at the critical state, when for every grain that is added to the pile, *on average* one grain will fall off the edge of the platform. However, the actual number of grains on the platform fluctuate continuously (Turcotte, 1992). For instance, at the critical state, the sand grains do not fall off the edge of the platform one at a time, but rather causing avalanches of any size. The size distribution for such avalanches gives a characteristic power law in the form of the hyperbolic relation:

$$N(S > s) \propto s^{-\tau} \quad (2)$$

where N is the number of avalanches with a characteristic size S above a threshold s, and τ is the scaling exponent.

In other words, the simple addition of grains has transformed the sandpile from a state in which the individual grains follow their own local dynamics to a stationary SOC state with its own emergent dynamics which could not have been anticipated from the properties of the individual grains (Bak, 1996). Notice that the sandpile is an open dynamical system characterized by a continuous energy flow. The critical state of the sandpile can be maintained only because energy is supplied by adding grains to the sandpile (uniformly, one grain at a time), and energy is dissipated (instantaneously) by grains falling off the edge of the platform. Notice also that at the critical state small avalanches and large avalanches are caused by the same mechanism "which can be understood only from a holistic description of the properties of the entire pile. Thus it appears that one should not try to come up with specific explanations for large avalanches, but rather with a general theory encompassing all avalanches, large and small" (Bak, 1996).

In this paper, analyzing 9164 wildfire records of the Regional Forest Service of Liguria (northern Italy) from 1986 to 1993 for agreement with the power-law behavior of Eq. (2), we suggest that the idea of SOC is particularly adequate for understanding the general laws underlying broad-scale wildfire occurrence.

2. A statistical theory for wildfires

Since wildfires are highly complex events involving climatic, vegetational, and human factors as well as local physical and topographic conditions (Whelan, 1995), the circumstances that determine frequency and extent of wildfires on a regional basis are rather scarcely understood. For instance, although considerable work has been carried out to obtain a comprehensive understanding of the interactions between vegetation, physical conditions and fire characteristics of a given region, an analytical model which quantitatively parametrize these interactions into a small number of mathematical equations is far from being accomplished. However, even if such a model could be made, it is likely that the system would be highly nonlinear without any simple way to predict emergent behavior (Bak, 1996).

Now, as asked by Nagel (1992): "Is there anything we can say about these systems from first principles without knowing about the 'microscopic' details of the problem?".

The obvious answer is that such nonlinear systems must be necessarily treated statistically without producing specific details. Furthermore, since the cumulative power law of Eq. (2) is generally taken as the most striking signature of SOC (Noever, 1993), an experimental check of Eq. (2) with regard to broad-scale wildfire occurrence would constitute a first evidence that wildfire dynamics is self-organized critical.

3. Study area

Based on 9164 records of the Regional Forest Service of individual wildfires from 1986 to 1993, we compiled a 8-year fire history of Liguria including for each event a field estimate (measured in hectares) of the burned area. Liguria extends along the Mediterranean coast from the French border on the west to the Gulf of Spezia on the east, embracing the southern side of the Maritime Alps and Apennines as well as a large part of the Po Valley flanks.

The area of Liguria is 5416 km^2. Most of the territory is mountainous or hilly with narrow strips of lower terrain along the coast and the lower part of the largest alluvial valleys.

Along the coast, due to the presence of the mountains providing protection against continental influxes from the Po Valley, the climate is typically Mediterranean with dry summers and mild winters. Conversely, in the higher inland areas and on the Po Valley side, the climate becomes increasingly continental with cold winters and rain mostly during spring and autumn increasing from west to east. However, due to the complex geographical characteristics, Liguria is subject to a strong micro-climatic variability which is reflected in the high plant biodiversity of the region.

Woodlands cover an area of 2833 km^2, equal to 52.3% of the territory. The vegetation of the area is distinctly Mediterranean up to a height of 500 m with evergreen scrub and vast woods of *Pinus halepensis* Miller and *P. pinea* L. Beyond this lies a belt of mixed woods, up to approximately 800 m, with *Quercus sp.* L., *Ostrya carpinifolia* Scop., *Fraxinus ornus* L., *Acer sp.* L., *Carpinus betulus* L. and *Castanea sativa* Miller. From 800 to 1500 m there are woods of *Fagus sylvatica* L. and *Larix decidua* Miller, and *Abies alba* Miller from 1500 to 2000 m. However, as in any region of long standing settlement, the natural vegetation of Liguria has been largely transformed by human activity, especially at lower altitudes. The population distribution is heavily conditioned by the morphology of the territory. Of the inhabitants 90% live along the coast. In contrast, the mountainous inland areas are characterized by a progressive abandonment of agricultural practices.

4. Methods and results

To relate the occurrence of wildfires in Liguria to their area, we used the log–log transformed hyperbolic relation:

$$\log f(S > s) = \log k - \tau \ \log s \tag{3}$$

where f is the frequency of wildfires (number of wildfires per year) with a characteristic size S above a threshold s, and $\log k$ is a constant which represents a measure of the regional level of overall fire occurrence. Mandelbrot (1982) noted that the fractal dimension D of the cumulative power law of Eq. (1) relates to the scaling exponent τ as:

$$\tau = D/2. \tag{4}$$

We hypothesize that both constants, k and $D/2$, may vary from region to region depending on the local conditions which influence fire occurrence, such as climate or vegetation type.

In the size range $0.0 > \log s > 2.0$, the data are in excellent agreement with Eq. (3) ($R^2 = 0.997$) taking $D = 1.446$, and $\log k = 4.249$ N year^{-1} (Fig. 1). It appears therefore that the cumulative distribution of wildfires in Liguria during the period 1986–1993 shows a power-law behavior over two orders of magnitude, which is a significant range of sizes. Conversely, the power law is not obeyed both for fires smaller than 1 ha and for fires larger than 100 ha (i.e. in both cases the frequency of wildfires is less than expected from the log–log regression). For this deviations we hypothesize two possible explanations. First, it is well established that restricted scaling regimes appear unavoidably for natural fractals. For instance, the range of magnitudes of natural fractals is always bound within a lower and upper cut-off imposed by the specific characteristics of the analyzed system (Avnir et al., 1997). Therefore, the observed restricted power-law behavior can be interpreted as a natural consequence of the large system's size rather than speaking to any anomaly in system's behavior (Noever, 1993).

Second, due to the very large number of wildfires with size < 1 ha, we suspect that not all small-sized wildfires were recorded by the forest service. Conversely, regarding very large fires, personal communications with colleagues have elicited the hypothesis that the upper cut-off of the power law may be related to the broad-scale landscape heterogeneity of the analyzed region. That is, since fire dynamics is influenced by fuel distribution across the landscape, above a certain threshold, fire spread may be limited by major irregularities in fuel distribution. Furthermore, since high-intensity wildfires have negative economic and social effects, we also hypothesize that the massive human intervention might partially reduce the fire spread over larger areas. However, this is a debatable argument since it is generally recognized that suppression efforts have a limited effect, especially on large fires (Hargrove, personal communication). Notice also that since our analysis is based on 8 years of data between 1986 and 1993, the statistics on large fires, with small frequencies and uncertainties in the burned areas are rather unreliable.

The observations summarized by the power law of Fig. 1 are the indications that the system has reached the self-organized critical state (Bak, 1996). In other words, it seems that, due to a continuous (solar) energy supply, the vegetation of Liguria evolving through thousands of years, has organized itself into a (critical) state in which the next wildfire can be anything from a simple plume of smoke to a large catastrophe. Again, as we saw for the sandpile model, at the self-organized critical state, large avalanches (wildfires) follow the same law as small ones without playing a special role. Notice that, due to the scale-invariant nature of the observed distribution, fractal statistics might also be used as an effective tool for wildfire hazard forecasting on a probabilistic basis. For example, based on Fig. 1, about 38 fires with sizes larger than 50 ha are expected in Liguria each year. However, although wildfire power-law distributions would be useful simply as an empirical means of regional wildfire hazard forecasting, it is now becoming clear that since vegetation evolution is fundamentally nonlinear, their applicability has a deeper theoretical basis.

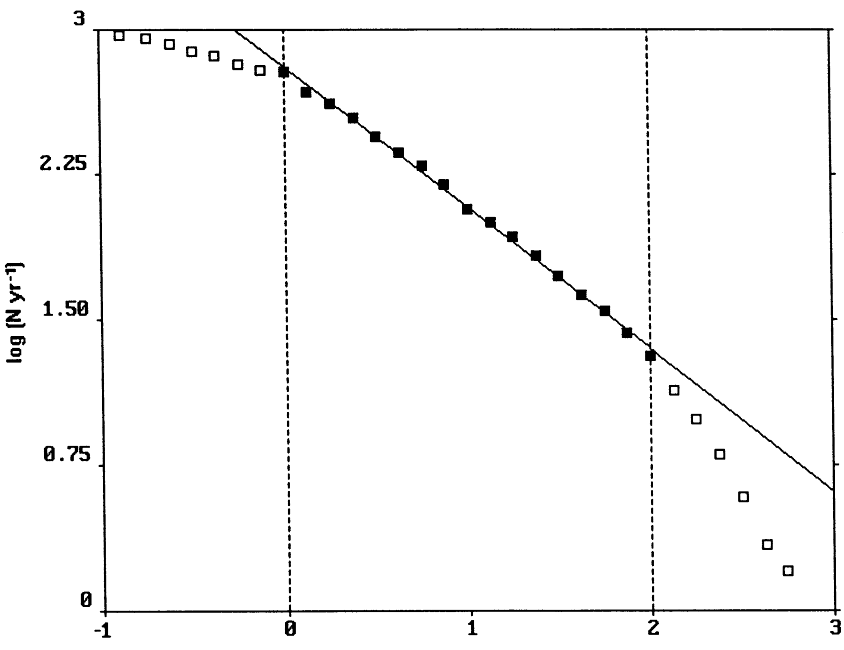

Fig. 1. Number of wildfires per year in Liguria during the 1986–1993 period, with size greater than s as a function of s. The regression line refers to the range $0.0 > \log s > 2.0$ (solid squares) taking $D = 1.446$ and $\log k = 4.249$ N year^{-1}.

5. Conclusion

Experimental evidence shows that the idea of self-organized criticality applies well to explain broad-scale wildfire occurrence. However, compared to the ideal sandpile model, the analysis of wildfires differs in two respects. First, unlike sieved sand, vegetation (fuel) distribution is not uniform and more closely approximate the kind of complex distributions seen in other probabilistic cellular automatons (e.g. Bak et al., 1990; Drossel and Schwabl, 1992). Second, in sandpiles, the avalanches start at the location where the slope first exceeds the local angle of repose. Conversely, more than 90% of wildfires in Liguria has human origin. The ignition mechanism of wildfires might therefore be more complicated, and even more interesting for ecologists, since it relates to the men's role in SOC models. We hope therefore that the present paper may constitute a first attempt to focus interest among ecologists for fractal statistics and SOC models.

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