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ANALYSIS OF ARCTIC MOUNTAIN VEGETATION: DIVERSITY AND VERTICAL DISTRIBUTION

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Abstract. The study is based on the data of botanical field work in nine mountainous regions of East Greenland between 71° and 81° northern latitude. Two methodological questions have been treated:
- how might the biological diversity of the vegetation of arctic mountains be best described?
- how the altitude of the vegetation-line might be estimated from field-observations?

The statistical analysis was based on the species/presence index. The best results were obtained using an index based on the slope of the cumulative species/log-abundance curve between the upper and lower quartiles of the distribution. For the estimation of the vegetation-line nonlinear regression methodology based on the logistic function is recommended.

1. Introduction

The discussion of recent climate change has lead to the question of how the vegetation of mountainous regions would respond to a rise of temperature during the growth-season. Such a change could cause multiple effects on the present pattern of the vegetation. Two questions are addressed:

- How does a change in the summer temperature influence the vertical distribution pattern of higher plants in the mountains?
- How does a change of climate influence the diversity of plant species in a mountainous region?

The altitudinal limits of plant species depend mainly on the temperature conditions during the growth season. The altitudinal limit of a given species might be used therefore as an indication to characterize the temperature during the vegetation period if the ecological requirements of the species are known. A change of temperature leads to a corresponding change of the altitudinal limits if the species has sufficient time to adapt the pattern of vertical distribution.

The diversity of a vegetation is in general estimated by an appropriate index of diversity calculated from the number and abundance of all species within a sample from a selected site or from a selected region. The value of an index of diversity changes if the number and/or the quantitative representation of the species change. It is assumed that an altering climate might influence the established ratio between dominant and rare species or the abundance of privileged or suppressed species in the vegetation. The analysis of diversity may be therefore a statistical tool to detect the influence of a changing climate on the vegetation.

2. Objectives and selection of data

Some methodological problems must be solved, before the questions about the influence of a changing climate on the altitudinal limits and on the diversity of a mountainous vegetation might be answered:

- How is the diversity of a mountainous vegetation described and which statistical methods may be used if the results of two or more field studies should be compared in terms of space or time?
- How should the altitudinal limit of vertical distribution for a plant species be estimated basing on the results of field studies?
- Which statistical methods may be used to compare estimated limits of vertical distribution for different regions and/or for different times?

These problems fit into the long-term program "Development and application of statistical methods in ecosystem research" running at the Department of Statistics, University of Dortmund (Germany) since 1984.

From the point of view of theoretical ecology it would be an advantage to base the study on data from mountainous regions without any direct impact on the vegetation by human activities. This well-founded condition was the reason to choose data from field-studies done by one of us in nine mountainous regions of eastern and northeastern Greenland done during the summers 1948-1952, 1954, 1956 and 1991.

3. Data and structure of data

3.1 Field work

The study is based on botanical field work during eight summer expeditions to East and Northeast Greenland. Most of the work was done in the summers 1948-1952, 1954 and 1956 when one of the authors (F.H. Schwarzenbach) was a member of the Danish East Greenland Expeditions lead by Dr. L. Koch and worked as a field-assistant in teams of two or three men lead by a geologist. At these times the teams were brought by motor-boats or by small sea-planes to their starting points at the coast (mostly the delta of a river) from where they established camps at higher levels (300-1700 m above sea-level) serving as a base for a series of excursions to key-points of the geological mapping. Camps have often been placed in the valleys on small terraces, on moraines or even on the ice of a glacier. Other camps were established on high plateaus at levels above 1000 m. During the geological excursions many peaks have been climbed.

Helicopters could be used only once for a five-days campaign (1956) to the isolated peaks (= nunatakker) near the edge of the Inland Ice at 74° N.

In 1991 a small botanical expedition on private base was organized to study the area of the Werner Bjerge at the southern side of the Staunings Alper. An airfield built 1957 (now abandoned) near the Pingo Pass (500 m) was used as a base for excursions within an area of about 100 km^2.

This traditional style of geological field work had advantages and disadvantages for the botanical studies. Observations and collections of plants could be made either at places, where the geologist had to work, or in the surrounding of the camps. As a consequence the geographical distribution of localities from where botanical observations are available shows a peculiar pattern. The sampling does by no means fulfil the recommendations of statisticians what must be kept in mind, when the results of the data analysis are discussed.

The results of the botanical observations are kept in the field-notes as list of the species of vascular plants found at a site covering mostly an area of a few m^2 when the vegetation was rather closed. The size of the covered area was enlarged up to 100-200 m^2 when the vegetation was sparse with isolated plants here and there. The list of plants was complemented with comments on the development (e.g. flowering, baring seeds) and the amount of the individual species as well with a detailed description and the ecological properties of the locality. Specimens of plants were collected at many sites. They are kept at the Greenland Herbarium of the Botanical Museum of the University, Copenhagen and have been identified by Danish and British botanists (K. Holmen, Th. Sorensen, T.W. Böcher, B. Fredskild, C. Bay and G. Halliday).

Whenever it was possible different types of sites where searched for to include as many types of vegetation as possible. The geographical position of the localities was pinpointed on aerial pictures if such pictures were already available at these times.

3.2 Organisation and structure of data

The botanical observations from the field-notes of the eight summers were transferred to a computerized database. A standardized form with 24 variables was used, coding the name of the species and information about the amount and the development of the plants. The number and the topographical position of the locality (northern latitude, western longitude, altitude) is given as well as the date of the observation. A short description of the site and the vegetation follows. Other variables indicate, when a herbarium specimen has been collected, who has identified the specimen and where it has been deposed. At least there are indications on photographies taken in the field and on aerial pictures showing the locality.

A record – called an "observation" – informs that the species in question has been found at a place for which the topographical position, the altitude in meters and the date is given together with a code marking all species found at the same place and included in the same plant-list. The code refers also the record to the field-notes.

The records are grouped to nine working-areas called "provinces" coded with "A" – "I". The provinces are subdivided in two or more "regions". The geographical position of the nine provinces is shown in Fig. 1.

The records of a province might be grouped either to "species" or to "localities" as it is shown in Table 1.

The four provinces A-D belong to the Staunings Alper (71-72° N), a mountainous area of about 2000 km^2 with an impressive scenery of rocks and glaciers with summits up to 2930 m (Dansketinde). From the centre of the Staunings Alper glaciated valleys run radially to all compass directions. The four provinces are defined as follows:

- The province "A" (Werner Bjerge) is situated in the south of the Staunings Alper at the eastern side of the immense Schuchert river draining the melt-water from many glaciers flowing in directions SE to SW.

Table 1. Number of observations, localities and species.

Province	Observations	Localities	Species
A	1493	94	126
B	582	45	98
C	606	43	92
D	693	43	94
E	1479	99	109
F	1145	81	120
G	932	48	119
H	1104	49	124
I	1314	118	98
A-I	9348	620	180

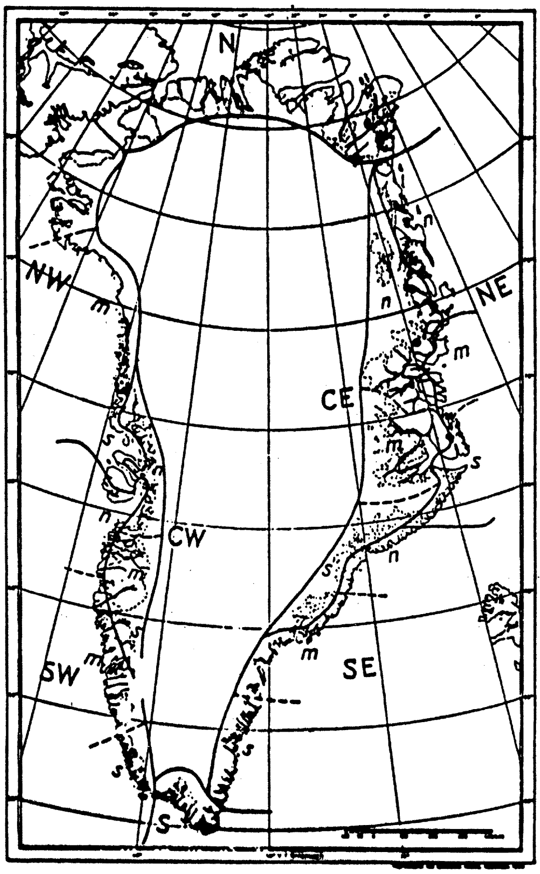

Figure 1. Map of Greenland.

- The province "B" covers the eastern side of the Staunings Alper and includes the valleys with rivers flowing into Kong Oscars Fjord.
- The province "C" includes the valleys of the Skjoldungebrae, the Linnés Gletscher and the Sedgewick Gletscher directed to the Segelsällskapets Fjord in the north of the Staunings Alper.
- The province "D" lies on the western side of the Staunings Alper. The localities belong to the basins of Vikingebrae, Gully Gletscher and Sefströms Gletscher merging into the Alpe-fjord.

The province "E" is situated west of the Staunings Alper with localities in the Schaffhauser Dal (West of Alpefjord), at the southern and northern coast of Forsblads Fjord and at the edge of an ice-shield W of Forsblads Fjord.

The southern part of Andrées Land (73° N) is called province "F", the northern part province "G". The working areas belong to a huge belt of Precambrian sediments. The two provinces are divided by the Grejsdal belonging to the province "G".

The province "H" (74° N) is characterized by two clearly separated regions. The first region included all localities in and around the Promenade Dal in Hudson Land. The second region in Ole Römers Land was visited during a five days helicopter-trip in 1956 landing on ice-free places ("Nuntakker") near the Inland Ice (Schwarzenbach 1961).

The province "I" is situated between 80° and 81° N in Kronprins Christians Land. There are three distinct regions: Centrum lake, northern coast of Ingolfs Fjord and Hekla Sund.

4. Species diversity and species presence indices

4.1 Community and sample species diversity measures

We consider a community of individuals categorized into s species with relative abundances π_i, so that $\sum_{i=1}^{s} \pi_i = 1$ and $0 < \pi_i < 1$ for $i=1,2,\ldots,s$. $\pi := (\pi_1,\pi_2,\ldots,\pi_s)'$ is called the *relative abundance vector* of the community. If $R(i;\pi)$ is the rarity of the ith species, Patil and Taillie (1979) and Gove et al. (1994) define the diversity of the community to be

$$\Delta(\pi) = \sum_{i=1}^{s} \pi_i R(i:\pi),$$

which is the mean value of the species rarity in the community. So the problem of defining a diversity index reduces to the problem of assigning a rarity value for each one of its species. If the rarity of the ith species depends only on its own relative abundance π_i, the diversity index then takes the form

$$\Delta(\pi) = \sum_{i=1}^{s} \pi_i R(\pi_i).$$

The rarity functions

$$R_1(\pi_i) = \frac{1}{\pi_i},\ R_2(\pi_i) = -\log\pi_i \text{ and } R_3(\pi_i) = 1 - \pi_i$$

provide the classical diversity indices:

$\Delta_i(\pi) = s$, (the *species count*),

$$\Delta_2(\pi) = -\sum_{i=1}^{s} \pi_i \log \pi_i, \text{ (the \textit{Shannon index}) and}$$

$$\Delta_3(\pi) = \sum_{i=1}^{s} \pi_i (1 - \pi_i), \text{ (the \textit{Simpson index}).}$$

The Shannon index ranges from 0 to log s and the Simpson index ranges from 0 to 1-(1/s).

In practice the diversity is measured for a sample drawn from the community, so it is important that any proposed index is independent of sample size, which is convenient for vegetation studies. This is achieved if the index is based on the species relative abundances.

Species abundance data are usually presented by reporting the number n_r of species with abundance r. So n_r is the number of species with r representations in the sample, where r=1,2,... .

If a sample of size N from a population contains S species with abundances N_i (i=1,2,...,S) estimates of $\Delta_2(\pi)$ and $\Delta_3(\pi)$ are given by

$$\hat{\Delta}_2(\pi) = -\sum_{i=1}^{s} \left(\frac{N_i}{N}\right) \log \frac{N_i}{N}$$

and

$$\hat{\Delta}_3(\pi) = 1 - \sum_{i=1}^{s} \frac{N_i(N_i-1)}{N(N-1)}.$$

To study the behaviour of the diversity sample statistics, we make assumptions about the form of the distribution of species abundances, which is given by $P(R=r)=n_r/S$ for r=1,2,... .

We assume that this species abundance distribution can at any instant be approximated by the lognormal or gamma-distribution.

An interesting approach to the measurement of diversity which takes into account the distribution of species abundances is the Q statistic, proposed by Kempton and Wedderburn (1978). This index is a measure of the inter-quartile slope of the cumulative species abundance curve.

The x-axis shows species abundance on a $\log_{10}$ scale while the cumulative number of species is given on the y-axis. R_1 and R_2 are the 25% and 75% quartiles, n_{R_1} is the number of species in the class where R_1 falls and n_{R_2} is the number of species in the class where R_2 falls.

For discrete data the index may be estimated by the sample statistic

$$\hat{Q} = \frac{\frac{1}{2} \cdot n_{R_1} + \sum_{r=R_{1+1}}^{R_2-1} n_r + \frac{1}{2} \cdot n_{R_2}}{\log \frac{R_2}{R_1}}$$

where the sample quartiles R_1 and R_2 are chosen such that

$$\sum_{r=1}^{R_1-1} n_r < \frac{1}{4} S \leq \sum_{r=1}^{R_1} n_r$$

and

$$\sum_{r=1}^{R_2-1} n_r < \frac{3}{4} S \leq \sum_{r=1}^{R_2} n_r,$$

where n_r is the number of species with abundance r.

If a single diversity index is required to characterize the pattern of abundances of species in a community Kempton and Wedderburn (1978) recommend Q in preference to the information measure or Simpson's index.

Q depends on a wide range of species with medium abundance, while the two other indices are strongly influenced by the abundance of the few commonest species. Also, Q shows smaller variability for samples taken from the same site in successive years and thus discriminates more clearly between different sites.

4.2 Species presence indices

According to Krebs (1989) the main objective of information theory is to try to measure the amount of order contained in a system.

Four types of information might be collected regarding order in the community:

(1) the number of species

(2) the number of individuals in each species

(3) the places occupied by individuals of each species, and

(4) the places occupied by individuals as seperate individuals.

In most community works only data of types 1 and 2 are obtained.

In our dataset we identify individuals with locality and in analogy to the notation in section 4.1 we define n_r as the number of species found at r locations in a study area and π_i is the proportion of localities, where species i was found. Using this notation we are able to define three species presence indices according to Simpson, Shannon and the Q-statistic.

4.3 Results

The results of our analysis are given in Table 2.

Shannon-Wiener-Index. The values of the Shannon-Wiener-index (H': basis log [e]) have an average of 4.218 ± 0.150 with a minimum of 4.025 (province B) and a maximum of 4.421 (H).

The values of the Simpson-index have an average of 0.980 ± 0.003 with a minimum of 0.974 (B) and a maximum of 0. 985 (G, H).

The values of the Q-index have an average of 31.768 ± 5.221 with a minimum of 24.650 (I) and a maximum of 38.715 (G).

The indices for the total of all 9348 observations in the nine provinces A-I have the following values: Shannon-Index = 4.510; Simpson-Index = 0.984; Q-Index = 39.033.

The Shannon-indices of the nine provinces and of the total are surprisingly high. The variability of the nine values is rather small, indicating that the diversity of the vegetation is similar within the provinces. This result has not been expected considering the differences of the ecological conditions (e.g. geology, continentality) and the distances between the nine provinces.

Simpson-Index. The Simpson-indices of the nine provinces have an average of 0.980 ± 0.003 with a minimum of 0.974 (B) and a maximum of 0.985 (G, H). The index for the total A-I has a value of 0.984.

The values of the Simpson-index are high and show a very low variability. The result confirms the statement from above that the nine provinces show a very similar degree of diversity. The Spearman rank correlation between the Shannon-Wiener-index and the Simpson-index has a correlation coefficient of 0.9167 (p=0.0095).

Table 2. Shannon-, Simpson-Index and Q-Statistics

Province	Observations	Species	Shannon	Simpson	Q-Statistic
A	1493	126	4.27	0.981	38.30
B	582	98	4.02	0.976	30.58
C	606	92	4.07	0.979	26.60
D	693	94	4.16	0.982	27.91
E	1479	109	4.19	0.981	28.74
F	1145	120	4.33	0.984	35.70
G	932	119	4.42	0.986	38.72
H	1104	124	4.42	0.986	34.73
I	1314	98	4.08	0.978	24.65
A-I	9348	180	4.510	0.984	39.03

Q-Index. The indices have a range between Q=24.650 (I) and 38.715 (G) with an average of 31.768 ± 5.221. The variability of the Q-index is much higher than the variability of the two other indices. The correlation coefficient (Spearman rank correlation) with the Shannon-Wiener-index has a value of 0.70000 (p=0.0477), with the Simpson-index of 0.5500 (p=0.11198), respectively.

4.4 Interpretation and conclusions

In spite of the low variability of the Shannon-Wiener-index and of the Simpson-index, the differences of the values might be explained to a certain degree by the specific ecological conditions of the nine provinces. If the three indices for the provinces are ranked, it is easier to find an interpretation.

The group of the four provinces A, F, G, and H have high indices indicating a well developed biological diversity of the vegetation. The province A lies farthest to the south and has an intermediate position between the Staunings Alper in the north and Jameson Land in the south. Jameson Land is known by its rich vegetation. The other three provinces are situated in the interior of the fjords (73-74° N) with favourable conditions (climate, geology, topography) for the growth of plants. Many types of vegetation are represented. The number of species ranges between 118 and 121. This first group of provinces seems to show the optimum of the arctic vegetation in the East Greenland mountains between 72° N and 76° N.

On the other hand the three provinces B, C, and I form a group with low indices indicating a vegetation with lower diversity. The province B at the eastern side of the Staunings Alper is situated along the coast of the broad Kong Oscar Fjord not far away from the outer coast. The weather during the summer is cool and wet. Fogs and temperature inversions are often observed. The mountain chain of the Syltoppene is built up by calcareous and dolomitic sediments offering poor living conditions.

The glaciers of province C flowing northwards have their origin in the central part of the Staunings Alper dominated by crystalline rocks. Species depending on calcareous soils are rare. Fens and wet heats are poorly represented. Due to the organisation of the geological mapping there are only a limited number of observations in the medium altitudinal belts between 300 m and 899 m. Compared with the provinces F, G, and H the variety of habitats and therefore the number of species is lower.

The province I between 80 and 81° N has a cold and dry climate with foggy weather and temperature inversions near the outer coast. Quite a few southern species have their northern limit south of 76° N not reaching the province I. Some parts of the studied area have a rather uniform vegetation poor in species.

The two provinces D and E belong to the continental belt between the Staunings Alper in the east and the Inland Ice in the west. Some species with southern distribution are found at isolated sites with favourable conditions even at altitudes up to 1300 m. The continentality and the steepness of the mountains restrict, however, the variety of habitats and therefore also the number of species.

The high values and the modest variability of the Shannon-Wiener-Index, the Simpson-Index and the *Q*-Index for the nine provinces in eastern and northeastern Greenland indicate that the diversity is near the optimum of a natural vegetation never disturbed by human activities.

The time passing since the latest big advance of ice in East Greenland has been long enough for the mountain vegetation to establish a well balanced pattern of the horizontal and the vertical distribution of the vegetation. The smaller advances and retreats of glaciers since the Ice Age – well documented by glaciologists – seem to have not influenced very much the diversity of the vegetation of these remote arctic regions.

This conclusion rises a few questions of high theoretical importance:

- If it is true that a vegetation under natural conditions reaches an optimal biological diversity, how is this optimum established and maintained during the longterm development of the vegetation?
- What is the reason that the "species/presence" distribution is always characterized by groups of common, less common and rare species, obviously following the same function?
- Is it possible to find an explanation for the surprising fact that the indices for a well consolidated natural vegetation are grouped around values below the maximum given by theoretical consideration?

Answers to these questions are wanted considering the claim of naturalists to preserve plant communities with a high degree of diversity and to restore plant communities with a poor diversity over-used by agriculture or by impacts.

5. Vertical distribution

5.1 Ecological importance of altitudinal limits

Botanists are interested to know which ecological factors control the development of plants and which conditions enable the existence of a species at a given place. Important factors are e.g. the average and the extremes of temperature as well as the length of the growth-season. In a mountainous region these factors depend on altitude: The average of the daily temperatures decreases with altitude. The snow melts later and the length of the snow-free season is reduced. There is a general experience that in the Swiss Alps the development of the vegetation is delayed by a week if the altitude rises 100 m.

A species depending on a long season with high temperatures for its existence prefers in Central East Greenland the lowland in the interior fjords with a rather continental climate. On the other hand a species requiring only a short summer to complete the annual life-cycle might be found in sheltered niches of rocks and screes at considerable altitude. The requirements of the different species concerning the temperature conditions during the growth-season vary within a broad range. As a consequence there is marked altitudinal gradient: The number of species decreases gradually with rising altitude. Not only the number of species but also the number of sites with suitable temperature conditions is reduced at higher levels. As the experience in the field has shown the number of observations for a given species decreases with higher altitudes.

The change of the temperature-conditions causes also the well-known sequence of altitudinal belts of vegetation characterized by the dominance of certain species. In East Greenland mountains the aspect of the dry heath of the lowland is dominated by the Dwarf Birch (*Betula nana*). At higher levels the aspect depends on the dominance of White Arctic Bell-heather (*Cassiope tetragona*) which might be recognized from far away by its colour.

The knowledge of the altitudinal limit for common and easily detected species offers the chance to the trained botanist to gain in the field an impression of the local conditions of temperature: The presence of a species is therefore a valuable indication for the temperature during the growth season. Mapping the presence of species botanists gain the valuable information about the temperature conditions depending on exposition, on the temperature-gradient from south to north or on the gradient of continentality from the outer coast to the interior of the fjords.

5.2 Estimated altitudinal limits of vascular plants

A simple method of estimating the altitudinal limit of a species is to start a botanical excursion from a point above the so called "limit of vegetation" (conventionally defined as the highest place where a vascular plant might grow). Walking downwards the altitude is noted for the first observation of each species. Obviously, the altitudinal limit of the species is as high or higher than the altitude of the site of the first observation. From the statistical point of view there are some doubts about this method based on such an extreme value.

It seems to be a better approach of the problem to base the estimation of the altitudinal level on the vertical distributions of all observations of a species. If the function of the vertical distribution is known the altitudinal limit of the species could be found by extrapolation.

To have a sufficient number of observations the statistical analysis of the problem of estimation the altitudinal limit of a species was replaced by the question: *How is the vegetation limit estimated for each of the nine provinces A-I, if all observations of all species in the province are included?* If this question can be answered the same approach might be used for the estimation of the altitudinal limit of the selected species.

5.3 Statistical method

For the estimation of the altitudinal limits of vascular plants in each province we plotted the number of observations which are observed at height x or below x. The height above sea level is reported in intervals of 50 m. The graph of this relationship between the number of observations and the height of occurence in each province looks sigmoidal. Therefore we fitted the logistic model

$$f(x) = \frac{\alpha}{1 + e^{-\kappa(x-\gamma)}},$$

where α, κ and γ are unknown parameters.

The parameter α characterizes the asymptotic behaviour of the graph, when the height variable x is increasing.

Using a least squares methodology (see Becka et al. 1993) we minimize

$$S(\theta) = \sum_{i=1}^{n} (y_i - f(x,\theta))^2,$$

where $\theta = (\alpha,\kappa,\gamma)$ is the vector of unknown parameters and n is the number of height-intervals considered in the specific province.

The vegetation line was determined by calculation of the 99%, 99.5% and 99.9% line of the asymptotic y= $\hat{\alpha}$. The height intervals corresponding to these horizontal lines were chosen as appropriate estimates for the vegetation-line.

5.4 Results

First, the estimation of the vegetation-line was based on the total number of observations for each province. In the second approach the obsevations have been ranked to altitudes and the values above the median have been used for the fitting of the curve. The latter approach has obviously lead to more convincing results, so the interpretation is based on these data.

Table 3 shows in column 2 the altitude of the highest locality of each province where a vascular plant still was found. Column 3 shows the highest points reached by observer in each province. Comparing these two columns it was decided if the vegetation-line has been passed during the excursions in each province. The vegetation-line has not been reached in province B, the vegetation-line in province H might be a little higher than the observed maximum at 1470 m.

Table 3.

Provinces	Highest observation	Maximal height reached by observer
A	1450	1670
B	1450	1450
C	2250	2600
D	1900	2900
E	2050	2200
F	1650	2200
G	1500	2200
H	1500	2050
I	1000	1400

Table 4. Estimates of vegetation-line.

	logistic curve					
	all observations			observations > median		
	99.0%	99.5%	99.9%	99.0%	99.5%	99.9%
A	1100	1150	1350	1350	1450	1750
B	900	1050	1300	1300	1500	1950
C	2750			1900	2050	2400
D	2400	2650		1900	2050	2400
E						
F	3150	3450	4200	1600	1750	1950
G	1500	1650	1950	1500	1650	2000
H						
I	1450					

Table 4 shows the estimates of vegetation-lines. Based on the observations above the median the vegetation-line could be estimated for six of the nine provinces. The provinces E, H and I have been omitted due to the heterogeneity of the material:

In province E the observations are aggregated at three levels: Many plants have been observed near the coast and an unusual high proportion of observations from 600-1000 m and from 1300-1550 m with scattered observation near 2000 m.

Due to the organisation of the field work the altitudinal distribution of observations shows some peculiarities influencing the results of the calculations: All observations of plants at each of the 19 landing-sites during the helicopter trip into the so-called "Nunatakker-region" (August, 14-18, 1956) of Ole Römers Land were grouped together. On the other hand the collection of data in the region of Vibekes Sö (230-780 m) was done in the usual manner.

The botanical work in the province I was done in three geographically separated regions. The vertical distribution of all observations show a distinct maximum at levels below 200m and a second peak from 550-680 m.

For the remaining six provinces the vegetation-line was estimated accepting the level of 99.0% of all observations.

There is a fairly good agreement between the estimated level of the vegetation-line and the highest observation of the vascular plant in the field with the exceptions of the provinces B and C. In province B there are only three localities above 800 m. The observations at higher levels are therefore underrepresented and the level of the vegetation-line seems

Table 5.

	A	B	C	D	F	G
Highest observation	1450	1450	2250	1900	1650	1500
Estimation	1350	1300	1900	1900	1600	1500
Differences	- 100	- 150	- 350	0	- 50	0

to be estimated to low. In the province C a single plant of the Bulbous Saxifrage (*Saxifraga cernua*) was found at an exceptional altitude near the summit of Elisabeths Tinde at 2220 m. The second highest observation was below 1800 m.

Conclusions

If the vegetation-line is estimated according to the described method (observation median) the results do correspond fairly well with the experience gained in the field; if the altitudinal distribution of the observations is not too irregular. As the analysis of the data from the provinces E, H and I show, the results are not convincing if the observations show a heterogeneous distribution between the coast of the sea and the vegetation-line. Therefore the botanist should try to study samples from all altitudes. It is recommended to split the interval between the coast and the vegetation-line into a series of 8-12 belts of the same range of altitude.

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