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SYNTAXONOMY: A SOURCE OF USEFUL FUZZY SETS FOR ENVIRONMENTAL ANALYSIS?

E. Feoli and V. Zuccarello, Dipartimento di Biologia Università degli Studi di Trieste, 34100 Trieste, Italia

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Abstract. The use of vegetation types defined in syntaxonomical research is proposed for environmental analysis on the basis of fuzzy set theory. An example with data of a relatively simple environmental system is discussed.

Introduction

Syntaxonomy may be considered as a source of fuzzy sets, being one of its aims the definition of vegetation types at different hierarchical levels (see Dierschke 1981). In fact each vegetation type consists of two sets of objects: one of species S found in the communities and the other of releves R on which the type is based. Since the same species may occur in many different types and the releves are assigned to types on the basis of the presence of species, or on the basis of a similarity level (polythetic assignment, Dale 1977) it would appear to be justifiable to attach a degree of membership to each species in S:

$$F\,(S) = \{\, s, f\,(s)\}$$

and to each relevé in R:

$$F\,(R) = \{\, r, f\,(r)\}$$

where s and r are respectively species and relevés and f is the degree of membership in a type.

In fuzzy set theory (Zimmerman 1985); introduced into plant ecology explicitly by Marsili-Libelli (1986), Roberts (1986) and Feoli and Zuccarello (1986), the degree of membership of the elements to a set ranges continuously between 0 and 1; while in traditional set theory it is either 0 or 1. In fact, the description of a vegetation type by the set of species and by the set of relevés is rarely a yes or no proposition, *i.e.*, categorically distinct, therefore the application of fuzzy set theory to community ecology seems particularly appropriate (Dale 1977).

In this paper the fuzzy set theory is applied to a relatively simple ecological system in order to demonstrate how the syntaxa can be used to explain trends of variations in relation to gradients and dynamics. Environmental fuzzy sets (EFS) are defined by chemical-physical data and used for ordering relevés of plant communities and species.

Data

A relatively simple syndynamic scheme of Giacomini and Pignatti (1955) representing the relationships between four vegetation types provided the data. The scheme is presented in Fig. 1. The releves corresponding to the types are presented in Table 1 (matrix **X** with 69 rows and 50 columns, rows = species, columns = releves). In this table the species of the two syntaxonomical classes: Salicetea herbaceae (including Polytrichetum and Salicetum) and Caricetea curvulae (including Hygrocurvuletum) are indicated by symbols, since only these species will be considered in the ordination scatter-plots. The average values for the measured environmental factors are presented in Table 2 (matrix **E** with 6 rows and 4 columns, rows = variables, columns = vegetation types). The syndynamic scheme on which the analysis is based has been simplified (p. 100, Fig. 11 in Giacomini and Pignatti 1955). Polytrichetum has been taken as a single type and Caricetum fuscae has been omitted (it shows only three species in common with the other types). The scheme of Fig. 1 rapresents a syndynamic series and reflects a gradient in which snow depth and snow persistence are the major controlling environmental factors. These both decrease from Polytrichetum to Hygrocurvuletum. For computations the Braun-Blanquet scale has been transformed according to van der Maarel (1979).

Method

The degree of membership of relevés in the types have been computed following the procedure suggested by Feoli and Zuccarello (1986). To obtain the environmental fuzzy sets (EFS) the stepp include:

1) The relevés in Table 1 are classified by sum of squares clustering based on geodesic distance (Orlóci 1978). This is comparable to the visual classification of Giacomini and Pignatti (1955).

2) Centroids are computed for each vegetation type according to Table 1 (matrix **X** 69×50). The centroids constitute the column vectors of the resulting matrix **C** (69×4).

	Community type n.	11111111111	2222222222222	33333333333333333333	444444
		11	1111111122222	22222333333333344444	444445
	Relevé n.	12345678901	2345678901234	56789012345678901234	567890
S1	Polytrichum sexangulare	++r54555543	2222211.+..+	.12+..1.+.+r.....111	......
S2	Anthelja juratzkana	443.+1+++++	.........3...	...........+r.......	......
S3	Webera commutata	2++++.+++++	....	....	......
S4	Webera carinata	.+..+...+..	....	....	......
S5	Kiaeria starkei	...+.+.....	....	...+....+..........+	......
S6	Pleuroclada albescens	.....+..+..	....	....	......
S7	Gymnomytrium varians	..2........	....	....	......
S8	Dicranum falcatum	.2.........	....	....	......
S9	Arenaria biflora	1+++2+2++++	+++.1+1.+....	.+1.11+++1.+........	......
S10	Gnaphalium supinum	+++11.12+++	222+232+21..2	122112+22121112++1+1	.....+
S11	Salix herbacea	23...+...++	5455554444544	...12++.....3.....++	++2++1
S12	Soldanella pusilla	.+.++......	+...+...+.+.+	+2+1+31+3+111112111+	......
S13	Sibbaldia procumbens	....+...+..	.++.+.+++1.+.	........+...+11..2+1	...1..
S14	Alchemilla pentaphyllea	....++.....	..+.+.+....21	55555535445535545454	......
S15	Veronica alpina	+.++++112++	1++.1+1.++..+	.....+..+1+.++....+.	......
S16	Gentiana bavarica	......++11+	....+........	..................+.	......
S17	Sedum alpestre	+..+....+++	11+++.+11....	.....+...1..+.......	......
S18	Sagina saginoides	+..1.....++	.......+.++.1	1.111.++.+1++.......	......
S19	Cerastium cerastioides	+1...+.+...	1+2.1++.1+.+.	..1..+....+.........	......
.	Chrysanthemun alpinum	+++1++22122	+1+12++1+1+2.	.2++12+1++1+++.+++.+	.1+1++
.	Poa alpina var. minor	++++.+.1+++	+1+12+++1++21	2+12121121+311112121	......
.	Taraxacum alpinum	+...+.+11+1	+++.+++......	+....+..++..+.......	......
.	Poa supina	..+.++1+..+	....	....	......
.	Ranunculus glacialis	..+..+++1..	....	....	......
C1	Carex curvula	.+.+.++...+	..++......+..	++1+.+1+1++.+2333444	344555
.	Cardamine alpina	+...1..++..	1++..++......	.....+..++..........	......
C2	Agrostis rupestris	...+...++..	.++.+.+.11..2	1+..3..+..2.++..+.+1	.+.1.1
.	Carex lachenalii	.+.....1...	....	....	......
.	Stereocaulum coralloides	...+...++32	....	....	+..+..
.	Ochrolechia sp.	.......11++	+.+1++..+1...	.++..1+.+.+..+++++++	......
.	Dicranoweisia crispula	.....++..++	....	....	......
.	Cladonia pyxidata	..+....2.+.	..+++........	..+..+..++++++++.+..	...+++
.	Cladonia rangiferina	........+++	....+........	.+...++........+++.+	+2.+++
.	Polytrichum piliferum	23	.......+.+.4.	.....+..++++.1+11+.+	..+..+
.	Pogonatum alpinum	....2......	....	....	...+.+
C3	Leontodon pyrenaicus	....	....	....1+++...+++++++++	1+1.1+
.	Polygonum viviparum	....	....+..+.....	.....+1.++.+2.+++.+.	11++21
C4	Phyteuma hemisphaericum	....	....	......+..+....+.+...	+...2.
C5	Ligusticum mutellina	....	.+...........	.......12223+2222211	1...12
C6	Potentilla aurea	....	....	........+...+1+++2+1	...2..
.	Sieversia montana	....	....	.........+..+.+++2++	...+..
C7	Nardus stricta	....	....	........+...++++...+	......
C8	Minuartia sedoides	....	.+++...1..+..	....	.+....
C9	Euphrasia minima	....	.1+3+.1.1.1.+	.....+.1+..+++..+...	....++
.	Euphrasia drosocalyx	....	.+.2..+1+..1.	............++..+..1	..+.++
.	Luzula spicata	....	...2...2.....	........1..+11....+..	++.++1
.	Luzula lutea	....	+....+.......	.........1....1.++...	......
.	Cardamine resedifolia	....	....1..1.....	..+.........+.......	......
.	Polytrichum juniperinum	....	.+.+2.+.+.++.	+.+++.1+.342+1+11++1	......
.	Cetraria islandica	....	..+....1...+.	1..1+.1+++...1++21.+	+3++12
.	Rhacomitrium ericoides	....	.......+.....	....+........3...+...	......
C10	Senecio carniolicus	....	....	....	+.++++
C11	Hieracium glanduliferum	....	....	....	.+....
.	Thamnolia vermicularis	....	....	....	.....+
.	Alectoria ochroleuca	....	....	....	....+.
.	Cetraria cucullata	....	....	....	...+..
C12	Veronica bellidioides	....	....	....	.++..2
C13	Festuca halleri	....	....	....	..+...
C14	Silene acaulis ssp. exscapa	....	....	....	.....+
.	Poa alpina	....	....	....	..221+
.	Oreocloa distica	....	....	....	331.41
.	Primula oenesis	....	....	....	.+1.++
.	Avenastrum versicolor	....	....	....	1.+..+
.	Soldanella alpina	....	....	....	....+.
.	Erigeron uniflorus	....	....	....	.+.+..
.	Phyteuma globulariaefolium	....	....	....	....1+
.	Cetraria aculeta	....	....	....	+2.+.+
.	Cetraria nivalis	....	....	....	+++..+
.	Paraleucobryum enerve	....	....	....	.....+

Table 1. Vegetation relevés used for computations representing the 4 types in Fig. 1. Codes for species: S1-19 (Salicetea), C1-14 (Caricetea).

3) **C** is multiplied by **X**:

$$C'X = Y$$

C and **X** are normalized by rows and columns. Because of normalization any element y (i, r) of **Y** (4×50) is cosine of the angle between a relevé r and the centroid of a type i, *i.e.* a measure of similarity.

The fuzziness of a relevé with respect to type i is computed by:

$$f(i, r) = y(i, r) - y(i \min) / (y(i \max) - y(i \min)).$$

4) Fuzziness of species with respect to vegetation types are computed by:

$$U = F X'$$

where **F** are normalized by columns and rows. **X** is normalized by rows. Also the values in **U** (4×69) are cosines and are transformed in order to obtain the fuzziness according to step 3).

5) Fuzziness of releves with respect to environmental factors (environmental fuzzy sets, EFS) are computed by:

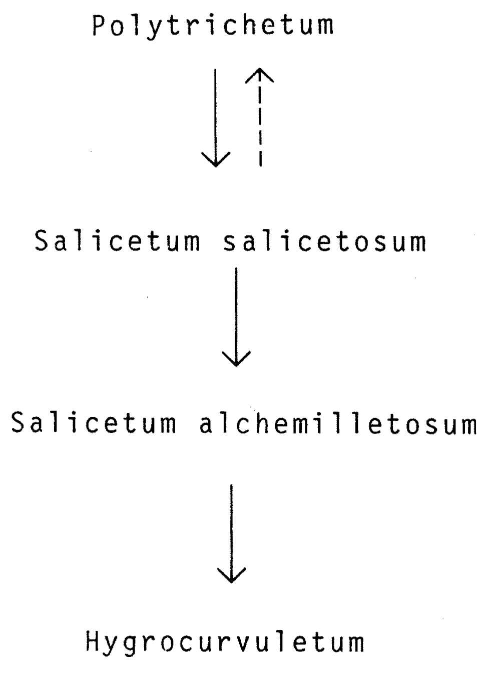

Fig. 1. A syndynamic scheme based on syntaxonomical data of alpine vegetation suggested by Giacomini and Pignatti (1955).

Table 2. Description of vegetation types by centroids of environmental variables.

	TYPE 1	2	3	4
pH	4.60	4.80	4.70	4.46
$CaCO_3$%	1.28	0.26	0.58	0.38
P_2O_5tot.%	1.44	1.43	1.83	1.73
P_2O_5sol.mg/l	0.05	2.50	3.10	5.03
Org.m.%	7.80	11.30	16.30	30.53
H_2O%	23.50	26.10	30.50	37.73
number ob.	4	3	13	3

$$A = E F$$

with **E** normalized by rows and **F** by columns. Also the elements in **A** (6×50) are cosines and are transformed to obtain measures of fuzziness to step 3).

6) Fuzziness of species with respect to environmental factors (EFS) are computed by:

$$Z = A X' \text{ or } Z = E U$$

with **A** normalized by columns and rows, **X** by rows, **E** by rows and **U** normalized by columns. Also the values in **Z** (6×69) are transformed as in step 3).

The row vectors of matrices **F**, **U**, **A** and **Z** can be used directly as ordination axes or by combining them two by two according to the anticommutative difference (Roberts 1986). For types i, h with respect to relevé r:

$$f(i/h)_r = (1 + (1 - f(h)_r)^2 - (1 - f(i)_r)^2) / 2.$$

The ordinations of relevés and species obtained by the same fuzzy sets may be superimposed (e.g., **A** on **Z**) to demonstrate the correspondence between species and relevés in the ordination scattergram. The Environmental Fuzzy Sets of **A** and **Z** have been both combined in a R-PCA (Orlóci 1978). The ordinations have been analyzed by the method of ellipses of equal concentration (Lagonegro and Feoli 1985a). Computations were carried out using the programs of Lagonegro and Feoli (1985b). Programs for computing fuzziness and anticommutative difference (FUZ and FUZZY) are available from V. Zuccarello.

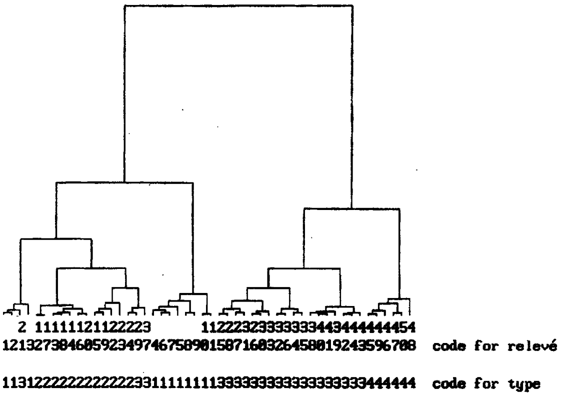

Fig. 2. Dendrogram of classification of the relevés of the 4 vegetation types in Fig. 1 and Table 1.

Results

The classification of the 50 relevés is presented in the dendrogram of Fig. 2. The ordination based on fuzzy sets corresponding to type 1 and type 4 (Polytrichetum and Hygrocurvuletum), which are the most dissimilar types, is presented in Fig. 3a. The sequence suggested by the syndynamic scheme is retained on both axes. The distance between the centroids of the types is significant in all the cases, also between types 2 and 3 which have an apparent overlap. In this ordination, types 2 and 3, which are subassociations of the same association (Salicetum herbaceae), are apparently more closely related than the other types. The ordination of the species of Caricetea curvulae and Salicetea herbaceae according to the fuzzy sets corresponding to types 1 and 4 is presented in Fig. 3b. It may be overimposed on Fig. 3a in order to demonstrate the correspondence between types and species. The centroid of Salicetea species is placed just between the centroids of types 1 and 2, whereas the centroid of Caricetea species is placed between the centroids of types 3 and 4. The ellipsis of Salicetea shows an overlap with the ellipses of types 1 and 2; the ellipsis of Caricetea species shows an apparent overlap with the ellipses of types 3 and 4. The ordination shows that among the species of Salicetea, *Gymnomytrium varians, Pleuroclada albescens, Antheljajuratzkana, Webera commutata, Webera carinata, Gentiana bavarica, Polytrichum sexangulare* are more related to Polytrichetum, whereas *Salix herbacea, Sedum alpestre* and *Cerastium cerastioides* are more related to Salicetum salicetosum and *Alchemilla pentaphyllea, Soldanella pusilla* and *Sibbaldia procumbens* are more related to *Salicetum alchemilletosum*. The ordination also shows that among the species of Caricetea, *Senecio carniolicus, Veronica bellidioides, Hieracium glanduliferum* and *Silene acaulis* are more related to Hygrocurvuletum, whereas *Ligusticum nutellina* and *Potentilla aurea* are more related to Salicetum alchemilletosum. Furthermore *Minuartia sedoides* is more related to Salicetum salicetosum. The ordination pattern of species reproduces graphically the pattern of the structured table in Table 1.

The ordination of releves and types based on anticommutative difference is presented in Fig. 4a, b. A

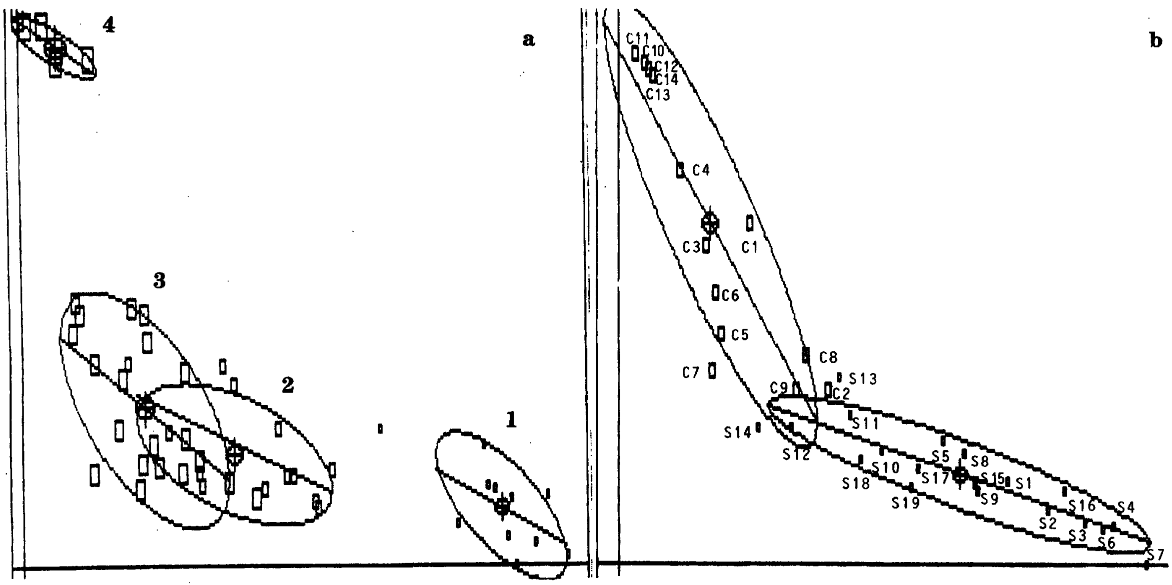

Fig. 3. Ordination of releves and vegetation types (a) and species (b) of Salicetea (S) and Caricetea (C) on the basis of fuzzy sets corresponding to type 1 (abscissa) and type 4 (ordinate). Ellipses of equal concentration for probability 0.05. Numbers 1-4 identify types as in Fig. 1 and Table 1.

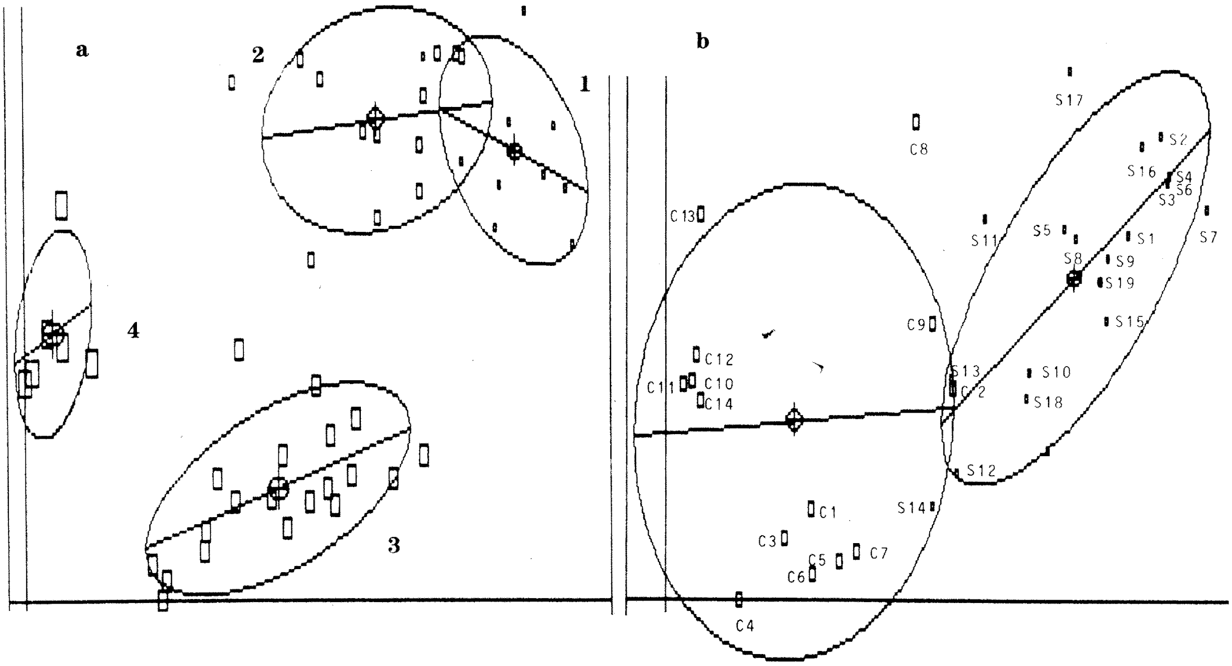

Fig. 4. Ordination of relevés and vegetation types (a) and species (b) of Salicetea (S) and Caricetea (C) by combining the fuzzy sets corresponding to types 1 and 4 (abscissa) and 2 and 3 (ordinate). Ellipses of equal concentration for probability 0.05. Numbers 1-4 identify the types as in Fig. 1 and Table 1.

clear separation of types 2 and 3 appears in Fig. 4a. The vertical axis is obtained by combining the fuzzy sets corresponding to these types. The sequence of the types in Fig. 1 is not retained along this axis because at its extremes only types 2 and 3 may be placed. This ordination offers a better resolving power of relevés and demostrates the strict relationship between types 1 and 2 suggested by the double arrows in Fig. 1. Also in this example the ordination pattern of species (Fig. 4b) reproduces graphically the one in Table 1.

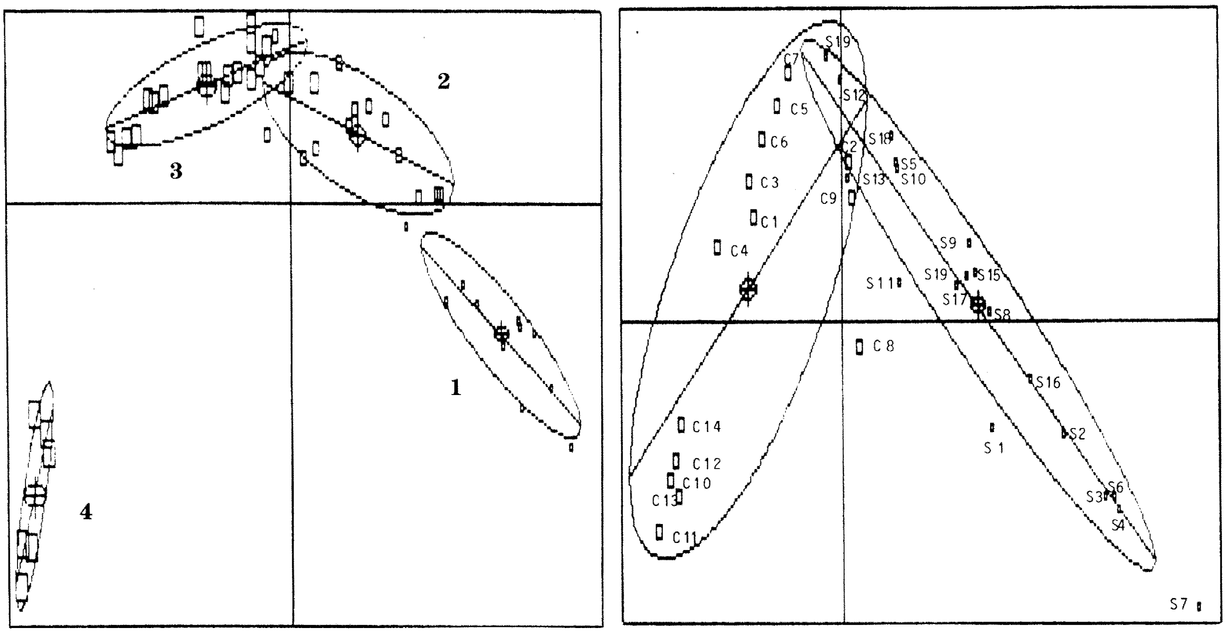

Fig. 5. Ordination of relevés and types (a) and species (b) of Salicetea (S) and Caricetea (C) according to PCA of environmental fuzzy sets. See the text for explanation. Ellipses of equal concentration for probability 0.05. Numbers 1-4 identify the types as in Fig. 1 and Table 1.

The ordinations based on environmental fuzzy sets (matrices **A** and **Z** combined in R-PCA) are given in Fig. 5 a and b. The correlation coefficients of EFS with the first components are given in Table 3. In both cases the first component explains a very high percentage of the total variance. Along the first component of Fig. 5a the sequence of the types follows that of Fig. 1. The centroids of the ellipses are all significantly separated. Fig. 5b shows the species of Salicetea well separated from those of Caricetea. Table 3 shows that the EFS are highly linearly correlated with the first two components in Fig. 5. From the correlation values of the EFS it follows that water content and organic matter are the environmental factors most related to the vegetational variation whereas pH is that least correlated.

Table 3. Correlation coefficients between the environmental variables and the first principal components (PCs) of the EFS matrices of relevés (rel.) and species (sp.).

PCs	pH	$CaCO_3$	P_2O_5t.	P_2O_5s.	O.M.	H_2O	EIGENVALUE	%
EFS rel.	0.591	0.875	-0.813	-0.968	-0.991	-0.997	4.69	78.14
EFS sp.	0.893	0.981	-0.829	-0.973	-0.982	-0.996	5.35	89.19

Conclusion

The Braun Blanquet approach (Braun Blanquet 1964; Westhoff and van der Maarel 1978) has defined a very large numbers of vegetation types in different regions of the world. The typification involved visual arrangements of phytosociological tables and/or numerical methods of classification and ordination. In many papers visually-defined types have been confirmed by numerical methods (e.g. Moore 1972, Werger 1973, Feoli and Lagonegro 1982). The present paper shows the high correspondence between the visual classification of Giacomini and Pignatti (1955) and the results of a clustering method. The numerical analysis suggests that the three relevés (1, 2, 3) assigned to the subass. dicranetosum by Giacomini and Pignatti (1955), being well separated from the other relevés of Polytrichetum, should actually be assigned to the Anthelietum described by others (see Giacomini and Pignatti 1955).

The importance of vegetation types in ecological work has been recognized and discussed in many papers and books (*e.g.*, Goodall 1963, Lieth 1968, Westhoff 1970, Whittaker 1970, Mueller-Dombois and Ellenberg 1974, van der Maarel and Werger 1978, Dierschke 1981). Personal opinions may differ (see McLean and Ivimey-Cook 1973, pp. 3431-3433), yet there is demonstrated need for hierarchical vegetation classifications. Ecological mapping at different scales and extrapolation of results obtained in the study of phytomass distribution and primary production could not be accomplished without such classifications. The present paper demonstrates on a simple data set that vegetation types can be used also as tools for explaining directly vegetation variation in relation to dynamics and gradients. They may be considered ''one step in a further study'' (Dale 1988) being the representatives of well defined clusters.

The use of syntaxonomical results in applications of fuzzy set theory reveals ordination patterns for species (variables) and relevés (syntaxonomical entities) along axes that are directly interpretable in terms of concepts or facts related to the biological composition of vegetation types, or in terms of environmental factors. This is an immense advantage over traditional ordination techniques and other methods based on eigenanalysis and non-metric multidimensional scaling (see Orlóci and Kenkel 1985). Another advantage is the possibility of carrying out direct ordination by environmental fuzzy sets notwithstanding the potential problem that environmental factors had not been measured in all the releves. The approach based on fuzzy set theory shows that in an area where a syntaxonomical study is available, detailed ecological analysis on the relationships between vegetation types can be done by a very simple computational procedure.

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