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ASSESSMENT OF DBH NORMALITY BASED ON PLOTLESS SAMPLING

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Abstract. A comparison between the use of Bitterlich relascope and the traditional method of basal area estimation shows that the probability distribution of tree diameters is much less non-normal than previously thought.

Introduction

Experience shows that the use of the Bitterlich relascope can render the procedure of basal area estimation more effective than the traditional method, which relies on areal sampling. However, important theoretical problems remain unsolved, which caution against the adoption of the Bitterlich and other similar methods, especially when estimating growth increments based on successive samples. A comparison between the two alternatives is suggested with the purpose to determine whether the advantages offered by the Bitterlich and the like methods are sufficient to offset the disadvantages. The comparison is concerned with the relative efficiency of two estimators $\hat{b}$ and $\breve{b}$, for plotless and plot sampling respectively, both circular and with a ϱ radius, related by

$$\frac{V(\breve{b})}{V(\hat{b})} = \frac{\int_0^\infty x^4 \, dF(x)}{\varrho^2 \sin^2 \alpha \int_0^\infty x^2 \, dF(x)}$$

In this expression $\hat{b}$ and $\breve{b}$ are estimating the basal area under the two alternatives, $V(\hat{b})$ and $V(\breve{b})$ are their variances, x is the diameter and F (x) is the distribution function of x.

The above relationship assumes that a theoretical model has been defined to fit the distribution of diameter frequencies. An elaboration of the model is the purpose of this paper.

Data

The data derived from a forest inventory of private wood-lands of the Veneto region, conducted during the summer of 1984. More than 28,000 trees were measured in 3,000 sampling areas of 48 different drainage basins each covering from 20 to 12,000 hectares. The trees measured had a minimal diameter of 2.5 cm. Tree selection in the field used the angle-count method with a basal area factor equal to 2.

Each basin was subdivided into homogeneous areal units. The *Picea excelsa* (Norway Spruce) was chosen for analysis in groups of 50 trees. Populations of this or larger size were found only in 46 units of 18 drainage areas. It is noted that hophorbeam *(Ostrya carpinifolia)* and beech *(Fagus sylvatica)* are the most widespread species of the region. But these were found in groups of at least 50 trees in only 15 and 25 units respectively. The general description of the 46 spruce samples is given in Table 1. With regard to the species selected, the origin of the trees is largely through natural regeneration. The regeneration is active. In all diametric categories the percentage of healthy trees is high. In most cases the inclination of the ground exceeds 45%.

Tests of joint normality

In order to test the 'joint' normality hypothesis of the 46 independent spruce samples, two different procedures were followed. The first is based on the third and fourth standardized moments (Table 2). Under the null hypothesis and following the normalizing transformations

$$X(\sqrt{b_1}) \text{ and } X(b_2)$$

where $\sqrt{b_1}$ and b_2 are the third and fourth moments, then

$$X^2(\sqrt{b_1}) + X^2(b_2) \simeq \chi^2(2)$$

and the significance can be tested in the usual way (D'Agostino and Pearson, 1973).

Furthermore

$$\sum_1^K \chi^2(2) \simeq \chi^2(2K)$$

where K is the number of samples considered.

Table 1. Sample (k), sample size (n), minimum and maximum diameter, mean diameter and standard deviation for each of the 46 samples examined.

k	n	min	max	mean	st
1	62	3.00	59.00	34.41	13.53
2	54	10.00	72.89	37.00	13.36
3	69	8.00	58.00	35.29	10.74
4	93	4.00	63.00	33.65	12.93
5	52	3.00	55.00	26.26	12.89
6	214	4.00	74.48	36.59	12.15
7	54	6.50	57.50	36.40	11.83
8	170	3.00	70.35	37.43	12.14
9	698	3.00	77.67	33.12	13.57
10	100	6.00	58.00	33.60	11.32
11	320	4.00	82.12	33.95	13.68
12	56	6.00	58.57	36.57	12.00
13	60	5.00	58.00	36.82	11.43
14	85	5.00	61.00	30.64	11.80
15	347	4.00	93.58	30.58	14.04
16	98	9.00	69.71	34.02	12.48
17	87	7.50	73.53	32.77	15.97
18	120	3.00	61.50	32.35	13.96
19	237	3.00	68.12	30.31	13.79
20	61	3.00	77.35	31.18	18.74
21	260	5.00	64.50	31.36	11.52
22	82	5.50	58.00	26.04	10.80
23	132	7.50	75.12	38.78	14.63
24	55	6.00	60.48	21.62	10.41
25	52	5.00	64.94	31.26	11.45
26	215	3.00	64.62	30.51	15.18
27	84	6.00	64.30	37.01	12.78
28	63	7.00	66.53	31.89	11.93
29	106	6.50	70.03	27.67	13.44
30	52	6.50	32.00	14.06	6.25
31	151	5.00	57.30	27.27	9.57
32	99	4.00	49.50	27.04	10.88
33	244	3.00	65.25	29.71	13.21
34	53	3.00	77.03	24.16	18.49
35	394	3.00	60.50	27.00	13.34
36	50	4.00	66.85	35.02	16.50
37	63	6.50	50.00	25.75	8.63
38	197	4.00	55.39	30.22	10.88
39	141	4.50	58.50	27.39	12.56
40	83	6.00	46.50	29.16	9.90
41	56	4.50	61.50	26.06	13.81
42	70	7.00	49.00	28.13	9.18
43	96	7.00	55.50	30.70	10.23
44	55	11.00	66.85	34.47	12.49
45	203	4.50	47.50	23.71	9.34
46	52	4.50	63.50	23.88	9.43

For the coefficient of skewness the D'Agostino-Pearson transformation has been adopted

$$X(\sqrt{b}_1) = \delta \sinh^{-1}(\sqrt{b}_1/\lambda)$$

The δ and $1/\lambda$ coefficients are calculated as a function of n. For kurtosis the equivalent normal standardized values are attained, according to Anscombe and Glynn (1983).

Table 2. Third ($\sqrt{b}_1$) and fourth (b_2) standardized moments of the samples.

k	$\sqrt{b}_1$	b_2
1	− .53	2.78
2	.13	2.85
3	− .44	2.95
4	− .08	2.35
5	.11	2.12
6	− .56	3.28
7	− .74	3.18
8	− .51	3.53
9	.02	2.60
10	− .40	2.57
11	.06	2.60
12	− .78	3.31
13	− .64	3.11
14	− .37	2.49
15	.41	3.75
16	.14	2.63
17	.60	2.96
18	− .22	1.81
19	.13	2.55
20	.31	2.27
21	− .04	2.58
22	.46	3.55
23	.02	2.42
24	1.77	7.22
25	.01	3.65
26	.11	2.21
27	− .62	2.70
28	.12	3.16
29	.93	3.89
30	1.26	4.23
31	.24	3.31
32	− .04	2.16
33	− .15	2.23
34	.93	2.94
35	.31	2.18
36	− .13	2.32
37	.35	3.76
38	− .24	2.58
39	.28	2.36
40	− .38	2.52
41	.31	2.48
42	− .29	3.05
43	− .13	2.55
44	.06	2.94
45	.10	2.40
46	1.09	7.88

If

$$A = 6 + \frac{8}{\sqrt{\beta}_1(b_2)}\left[\frac{2}{\sqrt{\beta}_1(b_2)} + \sqrt{1} + \frac{4}{\beta_1(b_2)}\right]$$

and

$$y = \frac{b_2 - E(b_2)}{\sqrt{V}(b_2)}$$

then

$$X(b_2) = \left[1 - \frac{2}{9A} - \left(\frac{1 - \frac{2}{A}}{1 + y\sqrt{2/(A-4)}}\right)^{1/3}\right] \backslash$$

$$\left(\frac{9A}{2}\right)^{1/2} \simeq N(0, 1)$$

with $E(b_2)$, $V(b_2)$ and $\beta_1(b_2)$

Table 3. Normalizing transformations of the third and fourth moments.

k	$X^2(\sqrt{b_1})$	$X^2(b_2)$	$X_1^2 + X_2^2$
1	3.184	.002	3.186
2	.186	.019	.205
3	2.467	.076	2.543
4	.113	2.558	2.671
5	.129	3.380	3.509
6	10.529	1.017	11.546
7	5.165	.485	5.650
8	7.197	2.153	9.350
9	.048	6.766	6.814
10	2.843	.707	3.550
11	.200	2.783	2.983
12	5.818	.782	6.600
13	4.358	.337	4.695
14	2.126	.981	3.107
15	9.411	5.679	15.090
16	.361	.404	.765
17	5.325	.065	5.390
18	1.056	41.942	42.998
19	.702	2.703	3.405
20	1.139	2.116	3.255
21	.073	2.482	2.555
22	3.106	1.603	4.709
23	.010	2.801	2.811
24	19.742	12.059	31.801
25	.001	1.692	1.693
26	.460	14.337	14.797
27	5.458	.126	5.584
28	.180	.434	.614
29	13.202	3.254	16.456
30	11.823	3.467	15.290
31	1.544	.994	2.538
32	.030	6.888	6.918
33	.958	15.151	16.109
34	7.472	.095	7.567
35	6.264	32.772	39.036
36	.174	1.196	1.370
37	1.429	2.141	3.570
38	1.974	1.746	3.720
39	1.960	4.196	6.156
40	2.192	.764	2.956
41	1.060	.518	1.578
42	1.125	.214	1.339
43	.306	.773	1.079
44	.040	.091	.131
45	.360	5.337	5.697
46	9.487	13.252	22.739

The second procedure adopted in testing the diameter normality avails itself for the W' statistic by Shapiro and Francia (1972) normalized according to Royston (1982) with

$$z = \frac{y - E(Y)}{\sqrt{V}(Y)} \qquad y = (1 - W')^{\lambda}$$

Table 4. The Shapiro and Francia's statistic (W' and normalized Z) and its significance.

k	W'	Z	sign
1	.955	1.637	.051
2	.984	−1.057	.855
3	.970	.700	.242
4	.970	1.022	.153
5	.957	1.219	.111
6	.955	4.301	.000
7	.940	2.200	.014
8	.969	1.941	.026
9	.975	3.804	.000
10	.957	2.231	.013
11	.976	1.669	.047
12	.944	2.071	.019
13	.957	1.468	.071
14	.935	3.352	.000
15	.973	2.678	.004
16	.978	.156	.438
17	.946	2.783	.003
18	.929	4.745	.000
19	.969	2.499	.006
20	.952	1.798	.036
21	.977	1.224	.110
22	..970	.841	.200
23	.975	.850	.198
24	.856	4.876	.000
25	.982	− .842	.800
26	.954	4.430	.000
27	.940	3.073	.001
28	.983	− .848	.802
29	.930	4.284	.000
30	.879	4.146	.000
31	.981	.041	.483
32	.965	1.567	.058
33	.956	4.590	.000
34	.867	4.502	.000
35	.948	8.265	.000
36	.950	1.579	.057
37	.975	.144	.443
38	.970	2.061	.020
39	.958	2.856	.002
40	.959	1.776	.038
41	.955	1.445	.074
42	.971	.575	.283
43	.979	.081	.468
44	.973	.178	.429
45	.970	2.110	.017
46	.911	3.209	.000

where λ, E (Y) and V (Y) are calculated as a function of n.

Observing Table 4, it can be inferred that for 26 samples the hypothesis of normality at a significance level below 5% is to be rejected. On the whole, the significance is very high, i.e. the standard normal variate is

$$\frac{\sum_{1}^{K} Z_i}{\sqrt{K}} = 14.48$$

It is worthwile noting that the 16 'significant' samples of the first test remain as such in the second test; the latter, however, shows a less conservative behaviour than the first, against all expectations (Pearson et al. 1977). Having established over-all normality, now we turn to examination of the effect of truncation. We say that a sample is truncated when it lacks all acceptable measurements larger or smaller than some limiting value and the number in the missing portion is unknown.

Truncation effect

Although during the research the threshold value of diameter was relatively small, below which trees were not considered (2.5 cm), the effect of the truncation turned out to be not negligible. The value of b_1 and b_2 of the complete distribution, shown in Table 5, have been calculated based on Glasser's (1967) formulae after 'correcting' the estimates of the truncated mean and variance by applying the function ϑ, as tabulated by Cohen (1961).

With an extremely small mean diameter (24.16 cm) and a large standard deviation (18.49 cm), the mean of the complete distribution results to be negative for the 34th sample; therefore it is not included in the table. However, the number of significant samples drops, as does the overall value of the test statistic

$$\sum_{1}^{45} \chi^2 = 232.77$$

which, nevertheless, remains highly significant (Table 6).

The greatest contribution, by far, is consistantly $X^2(\sqrt{b_1})$. This observation encourages investigations of the effect of relascopic sampling.

Relascopic sampling effect

Relascopic sampling is a form of PPS sampling. As the trees selected subtend an angle which is greater than a given angle α, their probability of being taken is approximately proportional to their diameters. The approximation is owing to the circle being only an idealization of the stem cross section shape. To restore the conditions of equi-probable sampling, the specimens were weighted with a linear function of their diameters and angular coefficient equal to the reverse of the plot radius factor which for a BAF = 2 amounts to 35.35 so that $\pi(x) = a - 35.35x$. Tables 7 and 8 show the results of the processing concerning these fictitious, equi-probable and complete samples. In spite of the joint statistic value being included in the 5% rejection zone

Table 5. Moments of the complete distribution.

k	b_1	b_2
1	.18	2.70
2	.11	2.77
3	.04	2.90
4	.16	2.71
5	.37	2.77
6	.07	2.84
7	.06	2.85
8	.06	2.86
9	.21	2.68
10	.21	2.82
11	.20	2.69
12	.07	2.85
13	.04	2.89
14	.17	2.70
15	.31	2.68
16	.13	2.75
17	.34	2.77
18	.26	2.66
19	.31	2.67
20	.04	3.92
21	.14	2.74
22	.25	2.62
23	.14	2.75
24	.40	2.70
25	.14	2.74
26	.35	2.81
27	.09	2.81
28	.15	2.72
29	.36	2.75
30	.51	2.35
31	.12	2.75
32	.22	2.64
33	.29	2.65
34	—	—
35	.37	2.79
36	.31	2.72
37	.10	2.78
38	.13	2.75
39	.33	2.66
40	.09	2.79
41	.35	3.08
42	.08	2.82
43	.08	2.81
44	.12	2.76
45	.23	2.62
46	.23	2.62

Table 6. Normalizing transformations of the moments of the complete distribution.

k	$X^2(\sqrt{b_1})$	$X^2(b_2)$	$X_1^2 + X_2^2$
1	2.071	.047	2.118
2	1.214	.000	1.214
3	.537	.028	.565
4	2.698	.132	2.830
5	3.543	.000	3.543
6	2.615	.097	2.712
7	.671	.021	.692
8	1.792	.029	1.821
9	23.114	3.909	27.023
10	1.499	.013	1.512
11	10.095	1.401	11.496
12	.735	.015	.750
13	.535	.033	.568
14	2.654	.138	2.792
15	16.588	1.721	18.309
16	2.294	.082	2.376
17	5.007	.038	5.045
18	5.209	.426	5.635
19	11.403	1.124	12.527
20	.459	2.636	3.095
21	5.904	.661	6.565
22	3.691	.321	4.012
23	3.190	.179	3.369
24	3.993	.033	4.026
25	1.393	.005	1.398
26	11.693	.164	11.857
27	1.392	.006	1.398
28	1.772	.030	1.802
29	6.285	.109	6.394
30	4.710	1.049	5.759
31	3.137	.232	3.369
32	3.860	.369	4.229
33	11.204	1.338	12.542
34	—	—	—
35	21.556	.660	22.216
36	2.933	.009	2.942
37	1.141	.003	1.144
38	4.230	.383	4.613
39	7.508	.580	8.088
40	1.467	.015	1.482
41	3.541	.289	3.830
42	1.033	.000	1.033
43	1.446	.013	1.459
44	1.304	.002	1.306
45	7.483	1.383	8.866
46	2.272	.112	2.384

Table 7. Moments of the weighted complete distribution.

k	b_1	b_2
1	.13	2.80
2	.02	2.72
3	.01	2.77
4	.07	2.63
5	.09	3.37
6	.03	2.71
7	.05	2.65
8	.02	2.72
9	.16	2.65
10	.04	2.67
11	.07	2.65
12	.06	2.65
13	.03	2.71
14	.10	2.63
15	.12	2.70
16	.03	2.69
17	.12	2.69
18	.14	2.88
19	.14	2.78
20	.55	4.03
21	.04	2.65
22	.08	2.57
23	.04	2.68
24	.06	2.56
25	.04	2.66
26	.07	3.43
27	.06	2.65
28	.04	2.66
29	.12	2.60
30	.11	2.27
31	.02	2.70
32	.12	2.57
33	.14	2.89
34	—	—
35	.14	3.02
36	.08	3.36
37	.01	2.77
38	.06	2.62
39	.15	2.69
40	.05	2.61
41	.02	3.77
42	.03	2.68
43	.02	2.71
44	.03	2.69
45	.10	2.53
46	.03	2.66

(but only by a narrow margin),

$$\sum_{1}^{45} \chi^2 = 118.94 \,, \; \chi^2_{.05}(90) = 112.86$$

the indication is clear that normality has to be rejected for 4 samples only.

Conclusion

The probability distribution of tree diameters at different ages, but of the same species, is much less non-normal than previously thought, provided that diameter determination is free from the effects of truncation and PPS sampling. Non-normality is mainly a consequence of skewness, thus bearing out the greater sensitivity of this particular feature compared to kurtosis.

Table 8. Normalizing transformation of the moments of the weighted complete distribution.

k	$X^2(\sqrt{b}_1)$	$X^2(b_2)$	$X_1^2 + X_2^2$
1	1.540	.000	1.540
2	.247	.014	.261
3	.164	.012	.176
4	1.165	.356	1.521
5	.935	.932	1.867
6	.962	.668	1.630
7	.574	.073	.647
8	.683	.395	1.078
9	9.634	4.911	14.545
10	.672	.282	.954
11	3.654	1.953	5.607
12	.644	.086	.730
13	.307	.038	.345
14	1.610	.305	1.915
15	6.613	1.408	8.021
16	.530	.195	.725
17	1.959	.157	2.116
18	2.886	.000	2.886
19	5.404	.323	5.727
20	5.694	3.010	8.704
21	1.976	1.478	3.454
22	1.205	.484	1.689
23	.955	.413	1.368
24	.583	.250	.833
25	.423	.056	.479
26	2.506	1.828	4.334
27	.987	.229	1.216
28	.497	.104	.601
29	2.198	.597	2.795
30	1.125	1.689	2.814
31	.534	.410	.944
32	2.070	.690	2.760
33	5.553	.023	5.576
34	—	—	—
35	8.771	.072	8.843
36	.738	.899	1.637
37	.121	.006	.127
38	1.974	1.288	3.262
39	3.694	.416	4.110
40	.830	.356	1.186
41	.254	2.107	2.361
42	.350	.107	.457
43	.354	.145	.499
44	.321	.035	.356
45	3.333	2.550	5.883
46	.306	.060	.366

Methods of computation

Data have been processed at the Centro di Calcolo, Università di Trieste, on a CDC 730. For the computation of W' and its significance level, Royston's (1982) routines have been used after some slight adjustments. The remaining programmes were written by the author in Fortran 5, calling some NAG routines. SPSS has also been used.

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