COENOSES 4 (1): 15-28, 1989

A COMPARISON OF PRINCIPAL COMPONENT AND FACTOR ANALYSIS AS ORDINATION MODELS WITH REFERENCE TO A DESERT ECOSYSTEM

S. Shahid Shaukat, Department of Botany, University of Karachi, Karachi-32, Pakistan
and Mudassir Uddin, Department of Community Health Sciences, The Aga Khan University Karachi-5, Pakistan

Keywords: Factor analysis, Factor loadings, Principal component analysis, Orthogonal rotation, Environmental ordination, Non-parametric multidimensional scaling

Abstract. Principal component analysis and three variants of factor analysis were tested as ordination models using environmental data of 22 stands from a desert ecosystem. The effect of orthogonal factor rotation was also examined. A variety of evaluative criteria were employed to characterize each of the environmental ordination. These criteria included: correlation between the ordination space and the original data space (mechanical validity), correlation between ordination axes and the environmental variables, correlation between the environmental ordination axes and components of PCA vegetational ordination, and the relationships between the environmental ordinations and the non-parametric multidimensional scaling vegetational ordination. With respect to the evaluation criteria multiple group and centroid methods, with and without axes rotation, gave best performance while the principal component ordinations with varimax and qartimax rotation performed poorly. The results are discussed in the light of the computational techniques involved.

Introduction

A host of formal and informal ordination techniques have been developed in the last three decades and these have often been reviewed (Whittaker and Gauch 1978, Noy-Meir and Whittaker 1978, Orlóci 1978, Gauch 1982, Greig-Smith 1983, Feoli 1977, Jolliffe 1986). Of the many ordination techniques proposed, principal component analysis has received most attention in ecological studies (Orlóci 1966, Gittins 1969, Bouxin 1975, Carleton 1980, Bradfield 1981, Gittins 1981). Ecologists generally use PCA as a descriptive tool for the purpose of trend-seeking though it often yields inferior results to those of reciprocal ordering (Gauch *et al.* 1977, Clymo 1980, del Moral 1980) or nonmetric multidimensional scaling (Fasham 1977, Kenkel 1984, Prentice 1977, Kenkel and Orlóci 1986, Bradfield and Kenkel 1987). However, Feoli (1977) and Nichols (1977) have pointed out that PCA can provide unique, objective and parsimonious representations that are predictable and ecologically meaningful.

Factor analysis (FA), which has been less frequently used in phytosociological studies than PCA, (Dagnelie 1965, Szöcs 1971), includes a group of related techniques that aim at the exposition of the underlying common covariance structure of a multivariate population (Harman 1976, Orlóci 1978, Jolliffe 1986). The fundamental difference between component analysis and factor analysis is that the former is a variance-oriented technique while the latter decomposes the covariance structure. Whereas the component scores in PCA are directly derived by a linear trasformation, the factor scores in FA are estimated by a variety of methods (Lawley and Maxwell 1971, Harman 1976, Seber 1984). Like the principal component analysis, the basic underlying model of factor analysis is linear. Thus, both the techniques are vulnerable to distortion when applied to non-linear data structures (Orlóci 1979). However, the problem of non-linearity and distortion of multidimensional relationships are largely circumvented when continously distributed suitably transformed environmental variables are used (Noy-Meir *et al.* 1975, Austin 1976, Rotenberry and Wiens 1980, Shaukat 1985, Jolliffe 1986).

The factor analysis routines involve the rotation of derived factors to a simple structure (cf. Thurstone 1947) which is expected to facilitate interpretation. This may be achieved through orthogonal or oblique rotations (Harman 1976, Srivastava and Carter 1983). Ecological studies have so far concentrated on orthogonal rotation by varimax procedure (Seligman 1973, Carleton and Maycock 1980). Carleton (1980), however, has tested the efficiency of oblique rotation.

A number of studies have compared various commonly used ordination methods (Gauch and Whittaker 1972, Austin 1976, Kessel and Whittaker 1976, Gauch *et al.* 1977, Fasham 1977, Roberston 1978, del Moral 1980, Okasanen 1983). The ordination methods have usually been evaluated using simulated coenoclines or coenoplanes (Kessel and Whittaker 1976, Fasham 1977, Gauch *et al.* 1981). A few comparative studies have employed field data (Robertson 1978, Clymo 1980, del Moral 1980, Okasanen 1983).

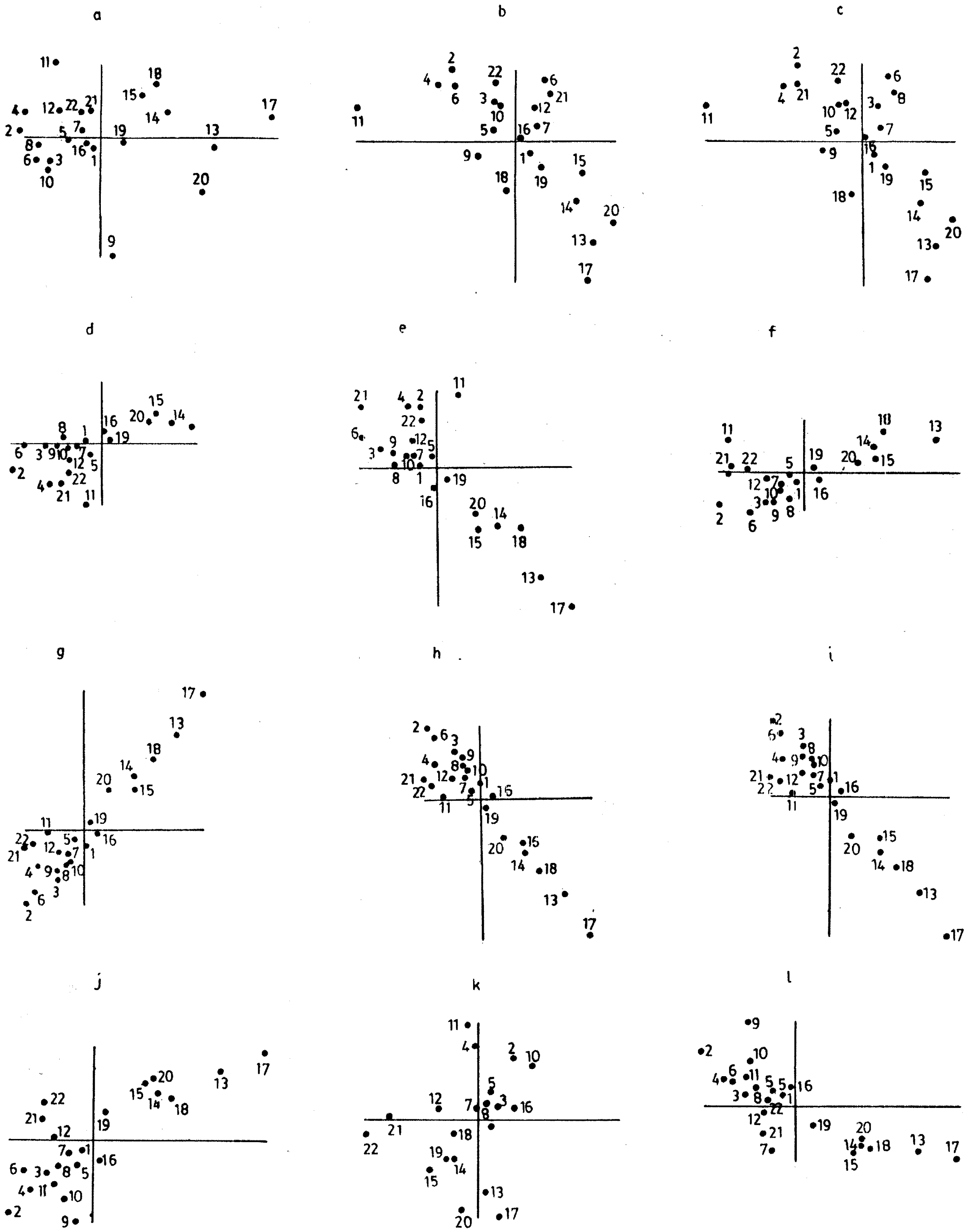

Fig. 1a-1. Two-dimensional environmental ordinations of 22 stands baseds on various methods. 1a, Principal component analysis (PCA); 1b, PCA with varimax rotation; 1c, PCA with quartimax rotation; 1d, centroid method; 1e, centroid method with varimax rotation; 1f, centroid method with quartimax rotation; 1g, diagonal method; 1h, diagonal method with varimax rotation; 1i, diagonal method with quartimax rotation; 1j, multiple group method; 1k, multiple group method with varimax rotation: 1l, multiple group method with quartimax rotation.

The purpose of this study is to compare the performance of component analysis and the different variants of factor analysis with and without factor (component) rotation with respect to a real ecological data set pertaining to a desert ecosystem.

Methods

The data set and its characteristics

The ordination methods were compared using the environmental data gathered from 22 stands in Gadap area, Southern Sind, Pakistan (Shaukat *et al.* 1980). This data set consists of 11 soil variables as follows: 1, soil depth (cm); 2, soil pH; 3, organic matter (%); 4, $CaCO_3$ (%); 5, exchangeable sodium (ppm); 6, exchangeable potassium (ppm); 7, maximum water holding capacity (%); 8, coarse sand (%); 9, fine sand (%); 10, silt (%); 11, clay (%). Environmental variables were used because these are monotic (James 1971) and the data matrix does not contain excessive zero entries. As opposed to vegetation variables (species), the environmental variables, to a great extent overcome the problem of non-linearity inherent in PCA and FA.

The environmental variables were transformed by simple transformations. Variables in percentages by arcsine transformation, while soil depth, K^+ and Na^+ to logits (Austin 1968). The corresponding vegetation data of the 22 stands was used to correlate the environmental axes (gradients) derived from FA and PCA. This data set was restricted to the importance value index (Curtis and McIntosh 1951) of 17 well represented species to avoid the problem of excessive zero values (Austin 1976). Furthermore, the data set was standardized to standard scores (Noy-Meir *et al.* 1975) for use in PCA.

Ordination methods

The environmental data set was subjected to principal component analysis (PCA) and three variants of factor analysis (FA), namely centroid, diagonal and multiple group methods (Fruchter 1954, Harman 1976). All these analyses used the correlation matrix which implies data centering and standardization to unit variance. PCA was performed using the package of Orlóci and Kenkel (1985) while the factor analysis solutions were obtained using the programs developed by the second author (M.U.). Three variants of FA were used namely, centroid, diagonal and multiple group methods. The algorithms used for these methods were those given in Fruchter (1954). The hypothesis of $k=3$ common factors gave the optimal solution (cf. Morrison 1976). Factor scores were obtained by the weighted summation technique suggested by Dale (1964). The PCA components and factor axes were rotated by varimax (VR) (Kaiser 1958) and quartimax (QR) (Neuhaus and Wrigley 1954) criteria for which programs were developed. The programs developed during this study are available from the authors on request.

Evaluation methods

Because no 'perfect', distortion-free ordination configuration of the data set can exist, a range of procedures were employed to evaluate the effectiveness of the ordination methods. The following evaluative procedures were used:

i) Correlation coefficient r (D, D*), coefficient of determination r^2 (D, D*), Euclidean distance Δ (D, D*) between the distance matrix of the environmental data set D and the corresponding matrix obtained for the first three components or the factor scores D* were computed. Correlation coefficient was used as a mathematical index rather than as a sample statistic. The r (D, D*) values are indicative of the mechanical validity of the ordination methods, i.e., how well the original p-dimensional relationships are expressed in a reduced t-dimensional ordination space.

ii) The ordination axes ere correlated with each of the 11 environmental variables to check how well the axes represented the environmental gradients.

iii) The three axes of the environmental ordination were correlated with the first three PCA components derived from the complimentary vegetational data set to evaluate the predictability of the ordination methods i.e., their ability to account for the variation in vegetational composition.

iv) Correlations were sought between each of the environmental ordinations and the nonparametric multidimensional scaling (NMDS) vegetational ordination. NMDS ordination was developed in accordance with Fewster and Orlóci (1983).

Results

The environmental ordinations

Fig. 1 shows two-dimensional ordination configurational based on principal component analysis (PCA), factor analysis (FA) with centroid, diagonal and multiple group methods and the rotated solutions of each using varimax and quartimax rotations. The ordinations, in general, show a continuity in habitat characteristics and discrete groups can not be recognized. Table 1 gives the percentage of total variance explained by the first three axes of each of the 12 ordinations. The highest proportion of total variance explained by the first axis was found for PCA (37.51%) (unrotated) followed by multiple group (unrotated ordination (34.86%) while the least proportion of total variance explained by the first axis was yielded by multiple group ordination with varimax rotation. The percentage of total explained variance by second ordination axis was lowest for unrotated PCA (18.64%) and highest for varimax rota-

Table 1. Percentage of total variance explained by the first three axes of each of the 12 ordinations.

Ordination method	Percentage of total variance explained: Axis 1	Axis 2	Axis 3	Cumulative
Principal component analysis	37.51	18.64	15.60	71.75
Principal component analysis with varimax rotation	23.63	35.53	20.79	79.95
Principal component analysis with quartimax rotation	23.43	35.38	20.13	78.94
Centroid method	34.00	28.97	27.38	90.35
Centroid method with varimax rotation	32.90	32.33	14.30	79.53
Centroid method with quartimax rotation	33.14	28.17	34.37	95.68
Diagonal method	31.72	34.16	18.86	84.76
Diagonal method with varimax rotation	31.87	34.02	17.91	83.80
Diagonal method with quartimax rotation	31.73	34.17	18.73	84.63
Multiple group method	34.86	29.49	28.94	93.29
Multiple group method with varimax rotation	12.82	31.56	32.91	77.29
Multiple group method with quartimax rotation	35.53	25.32	27.64	88.49

Table 2. Comparision of 12 ordination methods based on correlation coefficient and Euclidean distance between ordinations.

Method	PCA	PCA with QR	PCA with VR	CENT.	CENT. with QR	CENT. with VR	DIAG.	DIAG. with QR	DIAG. with VR	MULT.	MULT. with QR	MULT. with VR
PCA	0/1	0.7451	0.7461	0.7026	0.7046	0.7018	0.7225	0.7225	0.7220	0.7589	0.7592	0.7588
PCA with QR	235.81	0/1	0.9999	0.8962	0.8962	0.8957	0.8526	0.8527	0.8523	0.8134	0.8140	0.8140
PCA with VR	236.41	1.42	0/1	0.8951	0.8953	0.8946	0.8526	0.8527	0.8522	0.8150	0.8153	0.8153
CENT.	564.61	343.28	342.85	0/1	0.9995	1.0000	0.9561	0.9559	0.9559	0.8894	0.8896	0.8892
CENT. with QR	563.70	342.60	342.15	10.92	0/1	0.9995	0.9615	0.9613	0.9613	0.8966	0.8967	0.8963
CENT. with VR	564.77	343.54	343.11	1.12	10.57	0/1	0.9565	0.9563	0.9563	0.9563	0.8896	0.8809
DIAG.	492.83	278.70	278.19	115.45	110.47	115.25	0/1	1.0000	1.0000	0.9410	0.9410	0.9410
DIAG. with QR	492.34	278.16	277.66	115.89	110.96	115.69	1.04	0/1	1.0000	0.9411	0.9411	0.9409
DIAG. with VR	493.71	279.59	279.08	115.19	110/15	114.98	1.08	1.89	0/1	0.9411	0.9411	0.9410
MULT.	696.55	478.17	477.51	212.24	209.21	114.98	235.07	235.39	234.34	0/1	1.0000	1.0000
MULT. with QR	696.54	478.12	477.46	212.15	209.13	212.18	235.07	235.39	234.34	0.45	0/1	1.0000
MULT. with VR	696.88	478.39	477.74	212.57	209.53	212.61	235.07	235.71	234.66	0.89	1.10	0/1

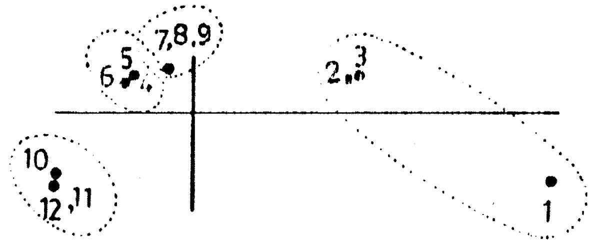

Fig. 2. Principal co-ordinate ordination of 12 environmental ordinations. The numbers 1 to 12 correspond to letters a to l of Fig. 1.

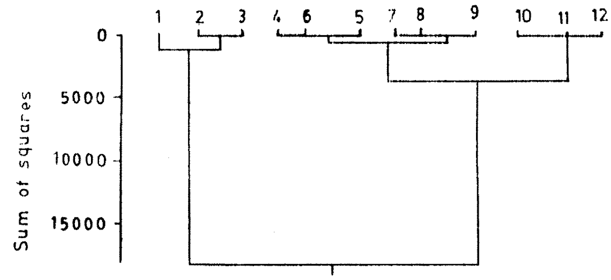

Fig. 3. Dendogram derived from sum of squares clustering of the 12 environmental ordinations based on Euclidean distance between ordination pairs. The numbers 1 to 12 correspond to letters a to l of Fig. 1.

ted solution of PCA (35.53%), though the second axis in diagonal method and its rotated solutions also gave high values of the proportion of total explained variance. Cumulative explained variance by the first three axes was highest for centroid method with quartimax rotation (95.68%) and lowest for unrotated PCA (71.75%). Rotation of axes in PCA caused an increase in the cumulative explained variance by the first three axes but a decrease in the multiple group method while diagonal ordination remained almost unaffected by rotation.

Comparisons of the 12 ordinations was performed by computing correlation coefficients and Euclidean distances between corresponding stand distances in the ordination space (Table 2). Correlation coefficients are invariably high indicating a great deal of similarity in the environmental ordinations derived from PCA and different variants of factor analysis. Ordinations derived from PCA and its rotated solutions were relatively less similar to those based on variants of FA and their rotated solutions compared to the similarity among the ordinations based on FA techniques. As expected, the ordinations based on rotated solutions of various factor analytical variants were identical to the original configurations (only the axes were rotated). However, the rotated solutions of PCA though closely similar were not identical to the unrotated PCA configuration (only the first 3 axes were used for rotation). The above results are exhibited in a condensed form in the principal co-ordinate ordination (Fig. 2) and the dendrogram derived from sum of squares clustering (Fig. 3) based on Euclidean distances between pairs of ordinations given in Table 2.

Table 3. Correlation coefficient r (D, D*), coefficient of determination r^2 (D, D*) and Euclidean distance Δ (D, D*) between ordination configuration and the origional environmental data set. (Three ordination axes were used).

Method	Correlation coefficient r (D, D*)	Coefficient of determination r^2 (D, D*)	Euclidean distance Δ (D, D*)	Explained variance (%)
Principal component analysis	0.77637	0.60276	416.33	60.27
Principal component analysis with varimax rotation	0.88828	0.78904	192.58	78.90
Principal component analysis with quartimax rotation	0.88809	0.78871	193.13	78.87
Centroid method	0.89370	0.79870	200.64	79.87
Centroid method with varimax rotation	0.89311	0.79765	201.11	79.76
Centroid method with quartimax rotation	0.89562	0.80213	200.00	80.21
Diagonal method	0.91673	0.84040	141.23	84.04
Diagonal method with varimax rotation	0.91649	0.83969	141.92	84.00
Diagonal method with quartimax rotation	0.91725	0.84134	140.58	84.13
Multiple group method	0.89082	0.79356	310.33	79.36
Multiple group method with varimax rotation	0.89091	0.79372	310.57	79.37
Multiple group method with quartimax rotation	0.89080	0.79353	310.33	79.35

Table 4. Correlation coefficients between the eleven environmental (soil) variables with the first three axes of PCA and FA environmental ordinations.

Ordination method	Variables	Axes of environmental ordination		
		Axis 1	Axis 2	Axis 3
Principal component analysis	Soil depth	0.3133	−0.1120	0.3748
	Soil pH	0.8699	0.3794	−0.9066
	Organic matter	−0.1100	0.3405	0.4720
	$CaCO_3$	0.7974	0.4585	−0.1776
	Exchangeable Na	0.7767	0.4423	0.0604
	Exchangeable K	0.8049	0.2496	−0.1733
	Max. water holding capacity	0.7055	−0.6324	−0.0801
	Coarse sand %	−0.4631	0.4797	−0.6843
	Fine sand %	0.0084	0.2374	0.8869
	Silt %	0.6783	−0.2431	0.0521
	Clay %	0.4476	−0.7512	−0.1242
Principal component analysis with varimax rotation	Soil depth	0.2255	−0.5161	−0.4752
	Soil pH	0.4221	−0.7676	−0.6487
	Organic matter	0.1142	0.0594	−0.2169
	$CaCO_3$	0.3035	−0.6968	−0.6072
	Exchangeable Na	0.3956	−0.7931	−0.9379
	Exchangeable K	0.3997	−0.7755	−0.5054
	Max. water holding capacity	0.5815	−0.6929	−0.1635
	Coarse sand %	−0.9336	0.4789	0.1101
	Fine sand %	0.5706	−0.0369	−0.2133
	Silt %	0.4206	−0.6049	−0.0478
	Clay %	0.4875	−0.5068	0.0571
Principal component analysis with quartimax rotation	Soil depth	0.2250	−0.5211	−0.4701
	Soil pH	0.4151	−0.7729	−0.6363
	Organic matter	0.1163	0.0491	−0.2224
	$CaCO_3$	0.2965	−0.7033	−0.5973
	Exchangeable Na	0.3905	−0.8105	−0.9323
	Exchangeable K	0.3927	−0.7742	−0.4890
	Max. water holding capacity	0.5804	−0.6791	−0.1419
	Coarse sand %	−0.9342	0.4652	0.0884
	Fine sand %	0.5710	−0.4010	−0.2097
	Silt %	0.4125	−0.5889	−0.0261
	Clay %	0.4906	−0.4946	−0.0398
Centroid method	Soil depth	0.4093	0.5173	0.5950
	Soil pH	0.8548	0.5415	0.5007
	Organic matter	0.0285	0.0674	0.0893
	$CaCO_3$	0.8060	0.4263	0.3862
	Exchangeable Na	0.9500	0.6500	0.6517
	Exchangeable K	0.7573	0.4409	0.3964
	Max. water holding capacity	0.4956	0.6530	0.5995
	Coarse sand %	−0.3258	−0.8029	−0.7798
	Fine sand %	0.1006	0.4021	0.4330
	Silt %	0.5315	0.6051	0.5773
	Clay %	0.2669	0.4732	0.4246

Tab. 4 - Continued

Ordination method	Variables	Axes of environmental ordination		
		Axis 1	Axis 2	Axis 3
Centroid method with varimax rotation	Soil depth	0.3300	–0.5142	–0.6756
	Soil pH	0.8828	–0.7030	–0.3657
	Organic matter	0.0105	–0.0580	–0.1393
	$CaCO_3$	0.8593	–0.6103	–0.3856
	Exchangeable Na	0.9599	–0.8199	–0.6979
	Exchangeable K	0.7956	–0.5974	–0.2906
	Max. water holding capacity	0.4082	–0.6126	0.2079
	Coarse sand %	–0.1482	0.6356	–0.2944
	Fine sand %	–0.0079	–0.2979	–0.0688
	Silt %	0.4648	–0.6029	–0.0433
	Clay %	0.1834	–0.4019	0.3271
Centroid method with quartimax rotation	Soil depth	0.4996	0.1911	0.3296
	Soil pH	0.7510	0.8497	0.8836
	Organic matter	0.0517	0.0079	0.0077
	$CaCO_3$	0.6682	0.8705	0.8440
	Exchangeable Na	0.8665	0.9082	0.9449
	Exchangeable K	0.6455	0.7815	0.7931
	Max. water holding capacity	0.5883	0.1702	0.4923
	Coarse sand %	–0.5638	0.1498	–0.2706
	Fine sand %	0.2538	–0.1063	0.0391
	Silt %	0.5920	0.2881	0.5209
	Clay %	0.3697	–0.0346	0.2634
Diagonal method	Soil depth	0.5694	0.3248	0.3714
	Soil pH	0.7479	0.8852	–0.0928
	Organic matter	0.0731	0.0119	0.4163
	$CaCO_3$	0.6729	0.8538	–0.0636
	Exchangeable Na	0.9101	0.9477	0.1542
	Exchangeable K	0.6443	0.7910	–0.2107
	Max. water holding capacity	0.5124	0.4542	–0.7429
	Coarse sand %	–0.4653	–0.2628	0.1678
	Fine sand %	0.2330	0.0697	0.6552
	Silt %	0.5293	0.5406	–0.5487
	Clay %	0.3017	0.2153	–0.7137
Diagonal method with varimax rotation	Soil depth	0.5817	–0.3357	–0.1133
	Soil pH	0.7373	–0.8816	0.0766
	Organic matter	0.0615	–0.0204	–0.4252
	$CaCO_3$	0.6589	–0.8486	0.0605
	Exchangeable Na	0.8970	–0.9515	–0.1932
	Exchangeable K	0.6383	–0.7850	0.1990
	Max. water holding capacity	0.5398	–0.4476	0.6955
	Coarse sand %	–0.4802	0.2709	–0.1025
	Fine sand %	0.2169	–0.0874	–0.6910
	Silt %	0.5438	–0.5350	0.5150
	Clay %	0.3318	–0.2090	0.6737

Tab. 4 - Continued

Ordination method	Variables	Axes of environmental ordination		
		Axis 1	Axis 2	Axis 3
Diagonal method with quartimax rotation	Soil depth	0.0596	−0.3244	−0.0408
	Soil pH	0.7480	−0.8853	0.0835
	Organic matter	0.0723	−0.0117	−0.4162
	$CaCO_3$	0.6723	−0.8542	0.0537
	Exchangeable Na	0.9103	−0.9480	−0.1658
	Exchangeable K	0.6446	−0.7913	−0.2030
	Max. water holding capacity	0.5132	−0.4536	0.7402
	Coarse sand %	−0.4641	0.2607	−0.1694
	Fine sand %	0.2307	−0.0680	−0.6531
	Silt %	0.5298	−0.5393	0.5436
	Clay %	0.3026	−0.2152	0.7131
Multiple group method	Soil depth	0.4685	−0.0337	0.1621
	Soil pH	0.8111	0.8796	0.8784
	Organic matter	0.0047	0.0268	0.1043
	$CaCO_3$	0.7483	0.8026	0.8644
	Exchangeable Na	0.8988	0.7383	0.9475
	Exchangeable K	0.7269	0.8053	0.7578
	Max. water holding capacity	0.5948	0.3818	0.1867
	Coarse sand %	−0.4384	−0.4315	−0.0749
	Fine sand %	0.1077	0.3300	0.1444
	Silt %	0.6031	0.4279	0.3044
	Clay %	0.3749	0.0836	−0.0456
Multiple group method with varimax rotation	oil depth	0.6310	−0.1079	−0.2575
	Soil pH	−0.4005	−0.7507	−0.9011
	Organic matter	−0.0986	0.0659	−0.6072
	$CaCO_3$	−0.3994	−0.6295	−0.8578
	Exchangeable Na	−0.1589	−0.5687	−0.9487
	Exchangeable K	−0.3567	−0.7224	−0.7947
	Max. water holding capacity	0.3373	−0.6744	−0.3817
	Coarse sand %	−0.1282	0.7396	0.2730
	Fine sand %	−0.3105	−0.3308	−0.1685
	Silt %	0.2118	−0.6209	−0.4530
	Clay %	0.5091	−0.3929	−0.1310
Multiple group method	Soil depth	0.3133	−0.1120	0.3748
	Soil pH	0.8699	0.3794	−0.0907
	Organic matter	−0.1100	0.3405	0.4720
	$CaCO_3$	0.7974	0.4585	−0.1776
	Exchangeable Na	0.7767	0.4423	0.6044
	Exchangeable K	0.8049	0.2496	−0.1733
	Max. water holding capacity	0.7055	−0.6324	−0.0801
	Coarse sand %	−0.4631	0.4797	−0.6843
	Fine sand %	−0.0084	0.2374	0.8869
	Silt %	0.6783	−0.2431	0.0521
	Clay %	0.4476	−0.7512	−0.1242

Mechanical Validity

Correlation coefficients, coefficient of determination and Euclidean distance between the distance matrix of the environmental data set and the distance matrices of resulting ordination configurations respectively denoted by r (D, D*), r^2 (D, D*) and Δ (D, D*) are presented in Table 3. These values indicate how effectively the ordinations represent the 11-dimensional relationships inherent in the environmental data set. Diagonal method and its rotated solutions gave the highest value of r (D, D*) followed by centroid and multiple group methods. PCA yielded the lowest value of r (D, D*) = 0.77637 but its quartimax and varimax rotated solutions (for the first three components) gave high values (0.88809 and 0.88827 respectively). Values for Δ (D, D*) varied inversely with r (D, D*). Percentage of explained variance was highest for the diagonal method of FA and its rotated axes ordinations.

Correlations between environmental variables and ordination axes

Correlation coefficients between the 11 environmental variables and the first three axes of the ordinations are set out in Table 4. With the exception of multiple group method with quartimax rotation and PCA with varimax and quartimax rotation, the first axis of ordi-

Table 5. Correlation coefficients between the first 3 components of PCA vegetational ordination and the first 3 axes of the PCA and FA environmental ordinations.

Ordination method	Axes of environmental ordination	Axes of vegetational ordination		
		Axis 1	Axis 2	Axis 3
Principal component analysis	Axis 1	0.6202	−0.0428	0.2353
	2	0.0926	−0.0779	0.2041
	3	−0.2119	0.2545	0.4265
Principal component analysis with varimax rotation	Axis 1	0.2468	−0.0331	0.2961
	2	−0.5593	−0.1274	−0.2051
	3	−0.4592	−0.1781	−0.1765
Principal component analysis with quartimax rotation	Axis 1	0.2425	−0.0315	0.3932
	2	−0.5633	−0.1322	−0.2053
	3	−0.4511	−0.1801	−0.1710
Centroid method	Axis 1	0.6129	0.0664	0.2582
	2	0.4026	−0.1595	0.3425
	3	0.3713	−0.2202	0.3555
Centroid method with varimax rotation	Axis 1	0.6286	0.0240	0.2070
	2	−0.5128	−0.1391	−0.3273
	3	−0.2397	−0.3892	−0.1695
Centroid method with quartimax rotation	Axis 1	0.5450	0.1255	0.3152
	2	0.5812	−0.0396	0.1321
	3	0.6333	0.0134	0.2292
Diagonal method	Axis 1	0.5400	0.1788	0.3027
	2	0.6251	0.0125	0.2545
	3	−0.2174	0.0529	0.1747
Diagonal method with varimax rotation	Axis 1	0.5391	0.1876	0.2966
	2	−0.6206	−0.0193	−0.2605
	3	0.2019	−0.0919	−0.1976
Diagonal method with quartimax rotation	Axis 1	0.5406	0.1790	0.3020
	2	−0.6254	−0.0120	−0.2535
	3	0.2102	−0.0541	−0.1768
Multiple group method	Axis 1	0.5948	0.1054	0.2688
	2	0.5367	−0.3016	0.2593
	3	0.5738	−0.0882	0.2312
Multiple group method with varimax rotation	Axis 1	−0.1209	0.5635	−0.0518
	2	−0.4872	0.2131	−0.2667
	3	−0.6072	0.0525	−0.2628
Multiple group method with quartimax rotation	Axis 1	0.6056	−0.0103	0.2782
	2	−0.4962	0.3560	−0.2260
	3	−0.5673	−0.0433	−0.2164

nations showed high positive correlations with soil pH, $CaCO_3$, exchangeable sodium, exchangeable potassium and percentage of silt. Coarse sand percentage was strongly negatively correlated with the first axis of the various ordinations with the exception of the varimax rotated solutions of centroid and multiple group methods of FA. The second axis was significantly correlated with $CaCO_3$ and exchangeable sodium in all the ordinations though sign of correlation coefficient differed. On the other hand, organic matter percentage and fine sand percentage were uncorrelated with the second axis. With respect to second axis the pattern of correlation of variables of the rotated solution of PCA was simillar to that of FA ordinations in particular that of centroid and multiple group methods while that of unrotated PCA was somewhat dissimilar to that of FA ordinations. The second axis of centroid method with quartimax rotation and multiple group method with quartimax rotation was correlated with only four variables: soil pH, $CaCO_3$, exchangeable sodium and exchangeable potassium. The second axis in other ordination methods was significantly correlated with at least 6 variables out of 11 with the exception of unrotated PCA. The degree of correlation between variables and the third ordination axis mostly varied with the ordination method. In most of the ordination methods exchangeable sodium showed significant correlation with the third axis while organic matter and coarse sand were usually uncorrelated. With regard to correlations of variables with the third ordination axis the rotated solutions of PCA and that of multiple group method showed close correspondence.

Correlation between environmental ordinations and PCA vegetational ordination

The first axis of the environmental ordinations yielded significant correlations with the first component of PCA vegetational ordination with the exception of rotated PCA solutions and varimax rotated multiple group method (Table 5). The second and third components of vegetational ordination were uncorrelated with the first axis of environmental ordinations with one exception. Only the first component of vegetational ordination gave significant correlation with the second and third axes of the environmental ordinations with a few exceptions. The second axis of unrotated PCA and that of centroid method of environmental ordinations was uncorrelated with the first, second and third component of PCA vegetational ordination (Table 5).

Correlation between environmental ordinations and NMDS vegetational ordination

Correlation coefficients computed between interstand distances in the NMDS vegetational ordination and the corresponding distances in the environmental ordinations are given in Table 6. The correlation coefficient were, in general, low though their corresponding t-values were high because of high number of paired observations ($n=231$). PCA environmental ordination gave the lowest correlation with the NMDS vegetational ordination while multiple group method gave the highest correlation. Rotation of axes in FA ordinations did not produce any marked change on the order of correlation. However, component rotation in PCA slightly increased the correlation coefficient.

Table 6. Correlation coefficients between NMDS vegetational ordination and each of the 12 environmental ordinations.

Ordination method	r	t
Principal component analysis	0.1257	1.9327
Principal component analysis with varimax rotation	0.1380	2.1288
Principal component analysis with quartimax rotation	0.1380	2.1288
Centroid method	0.1665	2.5914
Centroid method with varimax rotation	0.1616	2.5110
Centroid method with quartimax rotation	0.1504	2.3286
Diagonal method	0.1752	2.7352
Diagonal method with varimax rotation	0.1768	2.7617
Diagonal method with quartimax rotation	0.1752	2.7352
Multiple group method	0.2240	3.5688
Multiple group method with varimax rotation	0.2217	3.5283
Multiple group method with quartimax rotation	0.2235	3.5599

Critical value of t at $p = 0.05$ is 1.645.

The NMDS vegetational ordination was developed for a 2-dimensional solution with the initial configuration of PCA vegetational ordination. As a consequence the axes of NMDS were highly correlated with those of PCA vegetational ordination ($r=0.9740$ for first axis and $r=0.9170$ for second axis). It is worth mentioning that the axes of NMDS ordination do not have the same meaning as those of PCA which represent fitted lines through the data points. However, because of high cor-

Table 7. Correlation of first two axes of environmental ordinations and the two axes of NMDS vegetational ordination.

Ordination method	Axes of environmental ordination	Axes of vegetational ordination	
		Axis 1	Axis 2
Principal component analysis	Axis 1	0.6146	–0.1031
	2	0.0759	–0.1028
Principal component analysis	Axis 1	0.2668	–0.0334
with varimax rotation	2	–0.5418	–0.0819
Principal component analysis	Axis 1	0.2627	–0.0307
with quartimax rotation	2	–0.5450	–0.0875
Centroid method	Axis 1	0.5959	0.1293
	2	0.3982	0.1309
Centroid method	Axis 1	0.6100	–0.0332
with varimax rotation	2	–0.5011	–0.0988
Centroid method	Axis 1	0.5311	0.0816
with quartimax rotation	2	0.5564	–0.0934
Diagonal method	Axis 1	0.5168	0.1390
	2	0.6096	–0.0478
Diagonal method	Axis 1	0.5164	0.1475
with varimax rotation	2	–0.6044	0.0396
Diagonal method	Axis 1	0.5174	0.1391
with quartimax rotation	2	–0.6044	0.0478
Multiple group method	Axis 1	0.5800	0.0531
	2	0.5452	–0.3454
Multiple group method	Axis 1	–0.1435	0.5679
with varimax rotation	2	–0.5003	0.2555
Multiple group method	Axis 1	0.5970	–0.0625
with quartimax rotation	2	–0.5041	0.3949

relations with the corresponding PCA axes, correlation coefficients between NMDS vegetational ordination axes and those of environmental ordination can be gainfully examined (Table 7).

In general, the first axis of the environmental ordinations was highly positively correlated with the first axis of NMDS vegetational ordination with the exception of varimax rotated solution of multiple group method and rotated PCA ordinations. The magnitude and sign of other correlation coefficients varied with the ordination method.

Overall evaluation

Each method is rated in accordance with the evaluation criteria and average scores for ordination efficiency are given in Table 8. By the criteria employed, unrotated multiple group method and multiple group method with quartimax rotation were most efficient while PCA ordinations with varimax and quartimax rotation were least efficient. Centroid method with or without axes rotation was found superior to PCA and diagonal method.

Discussion and conclusions

This study has concentrated on the comparision of PCA and FA and the effects of orthogonal rotation of axes in these methods using a variety of evaluative criteria. The methods are examined in relation to an environmental data set pertaining to a desert ecosystem. On the other hand, previous studies have employed vegetational data sets either real or in the form of simulated coenoclines or coenoplanes. Both PCA and FA are based on linear models. Thus, the performances of such ordination methods are greatly affected by the curvilinearities inherent in the vegetation (spicies) data set because of non-linear Gaussian (presumably) species response along environmental gradients and the non-linear change of sample similarity with increasing dis tance between samples (Gauch 1973, Gauch *et al.* 1977). As opposed to vegetation data, the environmental data set comparises of continous uninterrupted variables that are mostly linearly correlated. Thus the problem of non-linearity and discontinuity is largely circumvented when an environmental data set is subjected to either PCA or FA.

One of difficulty in the use of FA, that has often been emphasized, is the calculation of factor scores (Blackith and Rayment 1971, Dale 1975, Greig-Smith 1983). A variety of formal methods have been developed to compute factor scores (cf. Harris 1967, Seber 1984, Mc

Table 8. Evaluation of environmental ordinations based on various evaluative criteria used. Ratings used are: 1, good; 2, fair; 3, poor.

Ordination method	Evaluative criteria						
	Explained variance		Mechanical validity	Correlation with PCA vegetational ordination		Correlation with NMDS vegetational ordination	Aver. score
	First axis	Cumulative*		First axis	Overall		
Principal component analysis	1	3	3	1	3	3	2.33
Principal component analysis with varimax rotation	2	3	2	3	2	3	2.50
Principal component analysis with quartimax rotation	2	3	2	3	2	3	2.50
Centroid method	1	1	2	1	2	2	1.50
Centroid method with varimax rotation	1	2	1	1	2	2	1.50
Centroid method withq uartimax rotation	1	1	2	2	1	2	1.50
Diagonal method	1	2	1	2	2	2	1.66
Diagonal method with varimax rotation	1	2	1	2	2	2	1.66
Diagonal method with quartimax rotation	1	2	1	2	2	2	1.66
Multiple group method	1	1	2	2	1	1	1.33
Multiple group method with varimax rotation	3	3	2	3	3	1	2.50
Multiple group meth od with quartimax rotation	1	2	2	1	1	1	1.33

* Cumulative % explained variance for first three axes.

Donald and Burr 1967). However, as Dale (1975) points out such formal procedures can be replaced with approximate methods. Dale (1964) showed that the weighted summation method, as a simple approximation, was an effective solution in practice. In the present study, weighted summation method for the computation of factor scores was employed with success, as the resulting ordinations effectively summarized the multidimensional relationships inherent in the original environmental data set.

The main disadvantage of FA over PCA relates to the choice of common factors. Several trials are often necessary to obtain a sufficiently close fit to the data. Convergence to an optimal solution was achieved for the hypothesis of $k=3$ common factors though the value of χ^2 was still significant. However, interestingly enough when 4 or more common factors were specified, either similar results were obtained to those yielded for $k=3$ common factors or the multidimensional relationships between variables/stands were distorted and rendered less interpretable.

The origional variables were, in general, more highly correlated with the derived variables (ordination axes) in FA than in PCA. This is the consequence of the fact that in FA, solutions for 3 common factors were obtained whereas in PCA only the first 3 out of 11 axes accounting for 71.75% of the total variation were retained. This is also the reason for the greater mechanical validity observed in the case of FA in contrast to PCA.

Orthogonal rotation of axes with varimax and quartimax criteria substantially improved the mechanical validity of PCA ordination but not that of FA ordinations. Furthermore, rotation of axes of the environmental ordinations did not render the axes ecologically more meaningful in terms of correlation with the vegetational ordinations.

In conclusion, it may be suggested that despite some numerical problems inherent in FA techniques that are not necessarily difficult to resolve, they can be used to develop more accurate and effective environmental ordinations than PCA.

REFERENCES

AUSTIN, M.P. 1968. An ordination of chalk grassland community. J. Ecol. 56: 739-757.

AUSTIN, M.P. 1976. On non-linear spicies response models in ordination. Vegetatio 33: 33-41.

BLACKITH, R.E. and R.A. REYMENT. 1971. *Multivariate Morphometrics*. Academic Press, London.

BOUXIN, G. 1975. Ordination and classification in the savanna vegetation of Akagera park (Rwanda, Central, Africa). Vegetatio 29: 155-167.

BRADFIELD, G.E. 1981. Component analysis of fire patterns in open eucalipt forest. Aust. J. Ecol. 6: 99-109.

BRADFIELD, G.E. and N.C. KENKEL. 1987. Non-linear ordination using flexible shortest path adjustment of ecological distances. Ecology 68. 750-753.

CARLETON, T.J. 1980. Non-centred component analysis of vegetation data: a comparision of orthogonal and oblique rotation. Vegetatio 42: 59-66.

CARLETON, T.J. and P.F. MAYCOCK. 1980. Vegetation of the boreal forests south of James Bay: non-centered component analysis of the vascular flora. Ecology 61: 1199-1212.

CLYMO, R.S. 1980. Preliminary survey of the peat-bog Hummel Knowe Moss using various numerical methods. Vegetatio 42: 129-148.

CURTIS, J.T. and R.P. MCINTOSH. 1951. An upland forest continuum in the prairie-forest border region of Wisconsin. Ecology 32: 476-496.

DAGNELIE, P. 1965. L'etude des communautes vegetales par l'analyse Statistique des liaisons entre les especes et. Les variables ecologiques: un example. Biometrics 21: 890-907.

DALE, M.B. 1964. The Application of Multivariate Methods to Heterogeneous Data. Ph. D. Thesis, University of Southhampton.

DALE, M.B. 1975. On objectives of methods of ordination. Vegetatio 30: 15-32.

FASHAM, M.J.R. 1977. A comparison of nonmetric multidimensional scaling, principal components and recipocal averaging for the ordination of simulated coenclines and coenoplanes. Ecology 58: 551-561.

FEOLI, E. 1977. On the resolving power of principal component analysis in plant community ordination. Vegetatio 33: 119-125.

FEWSTER, P.H. and L. ORLÓCI. 1983. On choosing resemblance measures for non-linear predictive ordination. Vegetatio 54: 27-35.

FRUCHTER, B. 1954. *Introduction to Factor Analysis*. D. Van Nostrand company, New York.

GAUCH, H.G. 1973. The relationship between sample similarity and ecological distance. Ecology 54: 618-622.

GAUCH, H.G. 1982. Multivariate Analysis in Community Ecology. Cambridge University Press, London, New York.

GAUCH, H.G. and R.H. WHITTAKER. 1972. Comparison of ordination techniques. Ecology 53: 868-875.

GAUCH, H.G., R.H. WHITTAKER and S.B. SINGER. 1981. A comparative study of nonmetric ordinations. J. Ecol. 69: 135-152.

GAUCH, H.G., R.H. WHITTAKER and T.R. WENTWORTH. 1977. A comparative study of reciprocal averaging and other ordination techniques J. Ecol. 65: 157-174.

GITTINS, R.T. 1969. The application of ordination techniques. In: I. H. RORISON (Ed.) Ecological aspects of the mineral nutrition of plants. Symp. Brit. Ecol. Soc. 1968, 9: 37-66.

GITTINS, R.T. 1981. Towards the analysis of vegetation succession. Vegetatio 46: 37-59.

GREIG-SMITH, P. 1983. *Quantitative Plant Ecology*, 3rd. edn. University of California Press, Berkeley, Los Angles.

HARMAN, H.H. 1976. *Modern Factor Analysis*, 3rd edn. Chicago University Press, Chicago.

HARRIS, C.W. 1967. On factors and factor scores. Psychometrika. 32: 363-379.

JAMES, F.C. 1971. Ordination of habitat relationships among breading birds. Wilson Bull. 83: 215-236.

JOLLIFFE, I.T. 1986. *Principal Component Analysis*. Wiley, New York.

KAISER, H.F. 1958. The varimax criterion for analytic rotation in factor analysis. Psychometrika 23: 187-200.

KENKEL, N.C. 1984. Boreal Vegetation at Elk Lake, Ontario: Types and Gradients. Ph. D. disert., University of Western Ontario, London, Ontario.

KENKEL, N.C. and L. ORLÓCI. 1986. Applying metric and nonmetric multidimensional scaling to ecological studies: Some new results. Ecology 67: 919-928.

KESSEL, S.R. and R.H. WHITTAKER. 1976. Comparison of three ordination techniques. Vegetatio 32: 21-29.

LAWLEY, D.N. and A.E. MAXWELL. 1971. *Factor Analysis as a Statistical Method*. Butterworth, London.

MCDONALD, R.P. and E.J. BURR. 1967. A comparison of four methods of constructing factor scores. Psychometrika 32: 381-401.

MORAL, R. DEL. 1980. On selecting indirect ordination methods. Vegetatio 42: 75-84.

MORRISON, D.F. 1976. *Multivariate Statistical Methods*. McGraw-Hill, New York.

NEUHAUS, J. and C. WRIGLEY. 1954. The quartimax method: An analytic approach to orthogonal simple structure. Brit. J. Statist. Psychol. 7: 81-91.

NICHOLS, S. 1977. On the interpretation of principal component analysis in ecological contexts. Vegetatio 34: 191-197.

NOY-MEIR, I., D. WALKER and W.T. WILLIAMS. 1975. Data transformations in ecological ordination. II. On the meaning of data standardization. J. Ecol. 63: 779-800.

NOY-MEIR, I. and R.H. WHITTAKER. 1978. Recent developments in continuous multivariate techniques. In: R.H. WHITTAKER (Ed.) Ordination of Plant Communities. Dr. W. JUNK Publ., The Hague, pp. 337-378.

OKSANEN, J. 1983. Ordination of boreal heath-like vegetation with principal component analysis, correspondence analysis and multidimensional scaling. Vegetatio 52: 181-189.

ORLÓCI, L. 1966. Geometric models in ecology. I. The theory and application of some ordination methods. J. Ecol. 54: 193-215.

ORLÓCI, L. 1978. Multivariate Analysis in Vegetation Research. 2nd edn. Dr. W. Junk Publs., The Hague.

ORLÓCI, L. 1979. Non-linear data structures and their description. In: L. Orloci, C.R. Rao and W.M. Stiteler (Eds.). Multivariate Methods in Ecological Work. Internation Cooperative Publ., Fairland, Maryland.

ORLÓCI, L. and N.C. KENKEL. 1985. *Introduction to Data Analysis with Applications from Population and Community Ecology*. Co-operative Publishing House, Fairland, Maryland.

PRENTICE, I.C. 1977. Nonmetric ordination methods in ecology. J. Ecol. 65: 85-94.

ROBERTSON, P.A. 1978. Comparison of techniques for ordinating and classifying old-growth floodplain forests in southern Illinois. Vegetatio 37: 43-51.

ROTTENBERRY, J.T. and J.A. WIENS. 1980. Habitat structure pathchiness, and avian communities in North American steppe vegetation: a multivariate analysis. Ecology 61: 1228-1250.

SEBER, G.A.F. 1984. *Multivariate Observations*. Wiley, New York.

SELIGMAN, N.G. 1973. A Quantitative Geobotanical Analysis of the Vegetation of the Golan. Ph. D. Thesis, Herbrew University, Jerusalem.

SHAUKAT, S.S. 1985. Approaches to the Analysis of Ruderal Weed Vegetation. Ph. D. Thesis, University of Western Ontario, London, Canada.

SHAUKAT, S.S., M.A. KHAIRI, D. KHAN and J.A. QURESHI. 1980. Multivariate approaches to the analysis of the vegetation of Gadap area, Southern Sind, Pakistan. Trop. Ecol. 21: 81-102.

SRIVASTAVA, M.S. and E.M. CARTER. 1983. An introduction to Applied Multivariate Statistics. North-Holland, N.Y. Amsterdam.

SZŐCS, Z. 1971. A Vértes-Hegység bükkösei: II. Interspecifikus korreláció-vizsgálatok. Bot. Kozl. 58: 47-52.

THURSTONE, L.L. 1947. *Multiple Factor Analysis*. University of Chicago Press, Chicago.

WHITTAKER, R.H. and H.G. GAUCH. 1978. Evaluation of ordination techniques. In: R.H. Whittaker (Ed.) Ordination of Communities. Dr. W. Junk Publs., The Hague.

Manuscript received: March 1988