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FUZZY SETS AND STRUCTURAL CHANGES IN FOREST SUCCESSION: AN EXAMPLE FROM BROAD LEAVED FORESTS OF N.E. ITALY

Enrico Feoli[1] **and Vincenzo Zuccarello**[2]

1 Department of Biology, University of Trieste, I-34100, Trieste, Italy
2 C. E. T. A., International Center for Theoretical and Applied Ecology, Gorizia Italy.

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Abstract. Fuzzy set theory is applied to test the coenocline model for structural successional changes in broad leaved forests of North-East Italy. As a working hypothesis, the test model used is the well-known successional sequence of *Ostrya-Quercus-Fagus*. The results show that fuzzy set theory allows to use a classification approach in a dynamic system perspective. The analytical structures recognized fit well the test model.

Introduction

Data for studying ecosystem dynamics may be obtained by direct long term observations on permanent plots (Beeftink, Daane and Munk 1971, Austin 1980, Hogeweg, Hesper, van Shaik and Beeftink 1985, Orlóci and Orlóci 1988, Roberts 1989a, Wildi 1988), by simulation models (Goodall 1967, 1975, 1977, 1981, Wildi 1978, Shugart, West and Emanuel 1981, Shugart 1984, Harrison and Shugart 1990), and by observations of communities living contemporaneously, but in different stages of a succession (Braun-Blanquet 1964, Feoli and Scoppola 1980). The latter approach is based on classification (Roberts and Morgan 1987) and was followed to develop the theory of succession through reasoning based on analogies (Cowles 1899, Weaver and Clements 1938, Schmidt 1969).

In the majority of the regions in the world, attempts to study the dynamics of natural forests on the basis of data from permanent plots, would be unrealistic. Yet the lack of suitable data is a severe drawback in the definition of parameters for simulation and in the validation of the models. For these reasons, we consider the classification approach as potentially useful in a dynamical system perspective (Roberts 1987, 1989b).

Given a set of static data, the problem to define a space with dynamical meaning can be faced by introducing into the data set the concept of succession. This can be done with the help of fuzzy set theory (Zadeh 1965, Negoita 1985, Zimmerman 1985). The application of this theory, as suggested by Feoli and Zuccarello (1986,1988), leads to definition of a new vegetation space. This is a fuzzy system space where the individual axes is interpretable as compounds of the factors, defining and controlling the existence of a given phenomenon defined by a fuzzy set. If the vegetation types are interpreted as vegetation states, their position in the fuzzy system space may be used to find possible trajectories from the early to the late successional stages.

A clear drawback of this approach is the lack of direct estimation of the transition times from one stage to another along the trajectories. This is the time necessary for the replacement of the different tree species in forest succession (see West, Shugart and Botkin 1981). Generally the estimation is based on the knowledge of the life history of the most important trees in determining forest structure and dynamics.

We apply fuzzy set theory to test if Whittaker's coenocline model (with the x axis representing time instead of an ecological gradient) would fit the life-growth forms along the well known successional sequence from dominant *Ostrya* to *Quercus* and to *Fagus*. This model appears justified for species (Beeftink, Daane and Munk 1971, Whittaker 1975, Hogeweg, Hesper, Schaik and Beeftink 1985, Roberts 1989a). However, it has never been tested for life-growth forms.

Data

Three data matrices describing 13 vegetation types from Poldini (1982) have been used:

1) Matrix **T** (15x13). This is the frequency data set by the main tree species (Table 1).

2) Matrix **L** (15x13). This contains relative frequency of the Mueller-Dombois and Ellenberg (1974) life-growth forms (Table 2).

3) Matrix **E** (8x13). This contains Landolt's (1977) average indicator values (Table 3).

Matrices **L** and **E** were obtained on the basis of the constancy classes of all the species in the synthetic tables of Poldini (1982). In this terms, life-growth forms have attached probabilities (x100) of finding tem in a type. This type of data has been discussed elsewhere by Feoli and Scimone (1985).

Methods

A set of entities with specified degrees of belonging to the set constitutes a fuzzy set (Zadeh 1965). There are many methods to compute the degree of belonging (Roberts 1986, Bezdek 1987, Dale 1988, Boryslawski and Krusinska 1989, Marsili-Libelli 1989). In the present paper the method uses the Feoli and Zuccarello (1986, 1988) computations based on matrices **L** and **T**. The 13 vegetation types have been classified into three groups corresponding to the three main successional stages in the broad leaved forests of NE Italy, characterized respectively by the high frequency of *Ostrya carpinifolia, Quercus petraea* and *Fagus sylvatica* (Feoli, Ganis and Poldini 1987). Centroids **C**(L) and **C**(T) of the three groups were computed for **L** and for **T**. With these matrices six types of fuzzy sets were obtained by matrix multiplication:

1) **C**(L) **L** = **F**(V;G)

2) **F**(V;G) **L'**= **F**(L;G)

3) **C**(L) **F**(V;G)= **F**(V;L)

4) **C**(T) **F**(V;G) = **F**(V;T)

5) **C**(T) **F**(L;G)= **F**(L;T)

6) **F**(L;T) **F**(L;T) = **F**(T;T)

These are matrices of degree of belonging values as follows:

1) **F**(V;G) of the 13 vegetation types to the three vegetation stages (G) of succession.

2) **F**(L;G) of the life-growth forms to the three vegetation stages.

3) **F**(V;L) of the 13 vegetation types to the sets defined by life-growth forms. The degree of belonging is conditional on the three main stages of succession. **F**(V;L) represents the link between life-growth forms and the vegetation types.

4) **F**(V;T) of the 13 vegetation types to the sets defined by the trees. Since this matrix is obtained by using matrix **F**(V;G), the degree of belonging is conditional on the vegetation structure and the three main stages of succession.

5) **F**(L;T) of life-growth forms to the sets defined by the trees.

6) **F**(T;T) of the trees to the sets defined by the trees. The degree of belonging is conditional on the vegetation structure and the three main stages of succession.

Before matrix multiplication, the variables were standardized and scaled to the interval (0,1). Fuzzy sets defined by *Ostrya* and *Fagus* in **F**(V;T), **F**(L;T) and **F**(T;T) were superimposed to order simultaneously vegetation types, life-growth forms, and trees.

Trajectories are defined by assuming that in the forest, during succession, light should decrease, while humidity, humus and nutrients should increase. To draw the trajectories from the early stages to the late stages of succession, the technique of kriging (Delfiner and Delhomme 1975, Hauser and Mucina 1991) has been applied to the space defined by *Ostrya* and *Fagus* fuzzy sets, using the indicator values (matrix **E**) of Landolt (1977). The kriging technique allows to visualize indicator value trends in the ordination scattergram defined by the fuzzy sets.

Matrix **F**(V;L) describes structural variation in the vegetation along the trajectories used to test if variation follows the coenocline model. This matrix is also used to measure the capacity of the life-growth forms to characterize structurally the states of vegetation in entropy terms according to De Luca and Termini (1972) and fuzziness as suggested by Yager (1979) (see Appendix).

Results

The ordination based on *Ostrya* and *Fagus* fuzzy sets is displayed in Fig. 1. Two possible trajectories are detected along two almost parallel curved lines. One is the vegetation types sequence 11, 2, 3, 5, 6, 7, 8 (trajectory (a)) and the other is the sequence 13, 12, 1, 10 (trajectory (b)). The two trajectories are characterized by regularly increasing humidity, nutrients, humus and decreasing light (Fig. 2). This regularity disappears if the points of the vegetation types were projected onto only one curve in the sequence 13,12,11, 9, 2, 4, 1, 3, 5, 6, 7, 8, 10. The two trajectories are respectively under and above the ideal line that would join the vegetation type with degree of belonging 0 to the *Fagus* fuzzy set and 1 to the *Ostrya* fuzzy set and the vegetation type with degree of belonging 1 to the *Fagus* fuzzy set and 0 to the *Ostrya* fuzzy set. The indicator values of humidity, nutrients, humus and dispersion are on the average higher on the first trajectory than on the second one (Table 4), while pH, light and continentality are higher on the second trajectory. Trajectory (a) identifies the succession on richer soil than that of trajectory (b). The vegetation types 4 and 9 are placed outside the two trajectories. The first one represents the most termophilic type in the *Quercus* stages in which the high temperature (see Table 3) prevents the development towards the *Fagus* stage, while the second one represents an intermediate stage in which the development would be prevented by ravine conditions where the type is confined (Poldini 1982). Fig. 1 shows that the life-growth forms and the trees may be grouped into two main groups, one (G1) with high degree of belonging to the *Ostrya* fuzzy set and the other (G2) with the same to the *Fagus* fuzzy set. G1 includes Chfrut., Chsuffr., Hcaesp., Pcaesp., NP, *Ostrya* *carpinifolia, Sorbus aucuparia, Pinus nigra, Pinus sylvestris, Sorbus aria, Fraxinus ornus, Quercus pubescens.* G2 includes Grhiz., Hrept., Chrept., Hros., Pscap., Plian., *Fagus sylvatica, Fraxinus excelsior, Acer pseudoplatanus, Tilia cordata, Carpinus betulus, Prunus avium, Acer campestre.* The early stages and the late stages of the succession appear well characterized both by trees as well as life-growth forms. However, the change between the early stages and the late stages are very gradual and not so clear if the original data are considered. By inspecting Table 5, presenting Table 2 rearranged according to the sequences of vegetation types along the two trajectories, only few of the life-growth forms show clear trends. Clearer trends of variation are detected if the degrees of the belonging of vegetation types to the sets defined by life-growth forms (matrix F(V;L)) are considered (Table 7). Almost all the life-growth forms show unimodal or monotonous trends, proving that the coenocline model of Whittaker (1967) fits closely the life-growth forms along the two trajectories. Only Hscap, Grhiz, Gbulb and Chfrut show slightly oscillation in the first part of trajectory (a).

The successional sequence *Ostrya-Quercus-Fagus* clearly appears along the two trajectories. From Table 7 the degree of belonging of the types to the *Ostrya* and

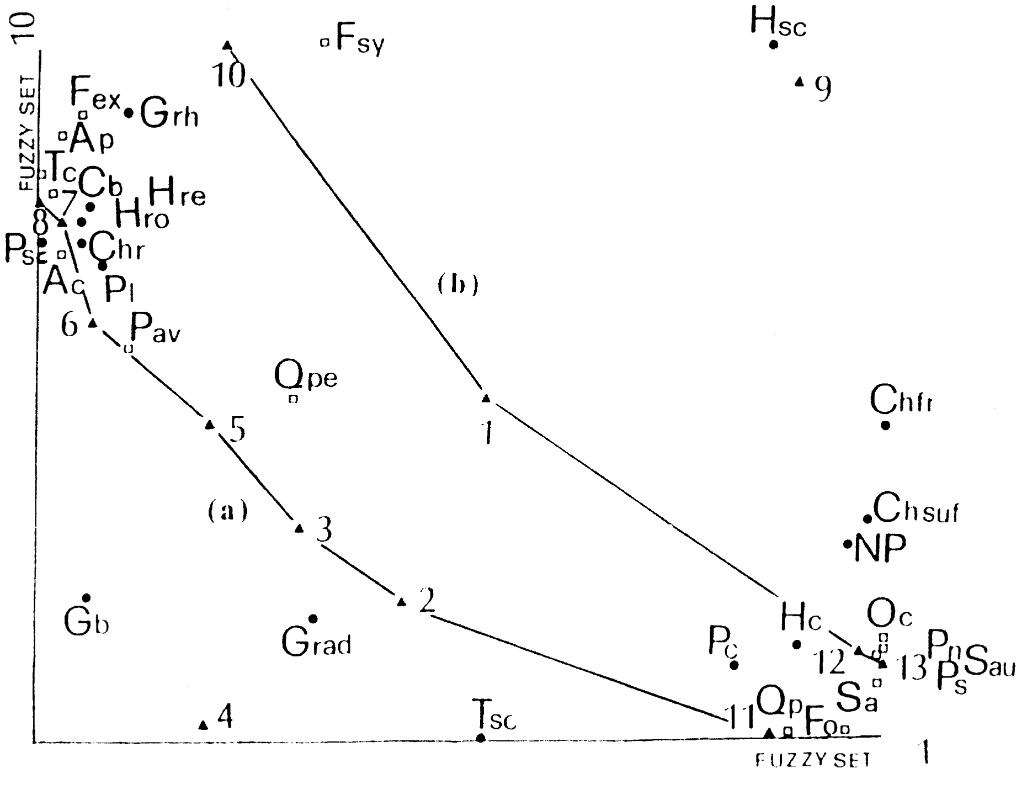

Figure 1. Ordination based on *Ostrya* (fuzzy set no. 1) and *Fagus* (fuzzy set no. 10) fuzzy sets. The triangles indicate vegetation types, circles growth-life forms, boxes the main tree species, (a) and (b) successional trajectories. Vegetation type codes: 1 = *Meruriali ovatae-Ostryetum;* 2 = *Buglossoido-Ostryetum capinifoliae polygaletosum*; 3 = *Buglossoido-Ostryetum capinifoliae hieracetosum*; 4 = *Carici-Quercetum petraeae violetosum*; 5 = *Carici-Quercetum petraeae quercetosum;* 6 = *Ornithogalo-Carpinetum betuli;* 7 = *Carpino-Fraxinetum excelsioris tilietosum;* 8 = *Carpino-Fraxinetum excelsioris cerastietosum*; 9 = *Hemerocallido-Ostryetum carpinifoliae;* 10 = *Ostryo-Fagetum*; 11 = Ass. *Betonica alopecuros-Ostrya carpinifolia;* 12 = *Orno-Pinetum ostryetosum*; 13 = *Orno-Pinetum nigrae pinetosum nigrae.* Growth-life form symbols: P = Phanerophytes; NP = Nanophanerophytes; Ch = Chamaephytes; T = Therophytes; G = Geophytes; H = Hemicryptophytes; l = lianas; re = reptant; fr = fruticose; suf = suffruticose; rad = radicigemma (root-budding); b = bulbosus; rh = rhizome; ro = rosulate; c = caespitose; sc = scapose. Main tree species symbols: Pn = *Pinus nigra,* Sau = *Sorbus aucuparia,* Oc = *Ostrya carpinifolia,* Ps = *Pinus sylvestris,* Sa = *Sorbus aria,* Fo = *Fraxinus ornus,* Qp = *Quercus pubescens,* Qpe = *Quercus petraea,* Pav = *Prunus avium,* Ac = *Acer campestre,* Cb = *Carpinus betulus,* Tc = *Tilia cordata,* Ap = *Acer pseudoplatanus,* Fex = *Fraxinus excelsior,* Fsy = *Fagus sylvatica.*

Fagus fuzzy sets may be expressed by monotonous functions while the degrees of belonging to the *Quercus* fuzzy set follow a unimodal function (Fig. 3). It is clear that types 2, 3, 5 and 6 of trajectory (a) and type 1 of trajectory (b) are closely related to the *Quercus* stages, that type 11 of trajectory (a) and types 13 and 12 of trajectory (b) are closely related with *Ostrya* stages, and that types 7 and 8 of trajectory (a) and type 10 of trajectory (b) are closely related to the *Fagus* stages. These two trajectories indicate that the succession from the *Ostrya* stages to the *Fagus* stages may follow two independent ways characterized by two main different edaphic conditions: one on poor soils (b) the other on more rich soils (a) (see Table 4). Along these two trajectories structural variation follows the same coenocline model (see Table 6). However, the intensity of the links of the vegetation types with the life-growth forms classes is different. The fuzziness of the fuzzy

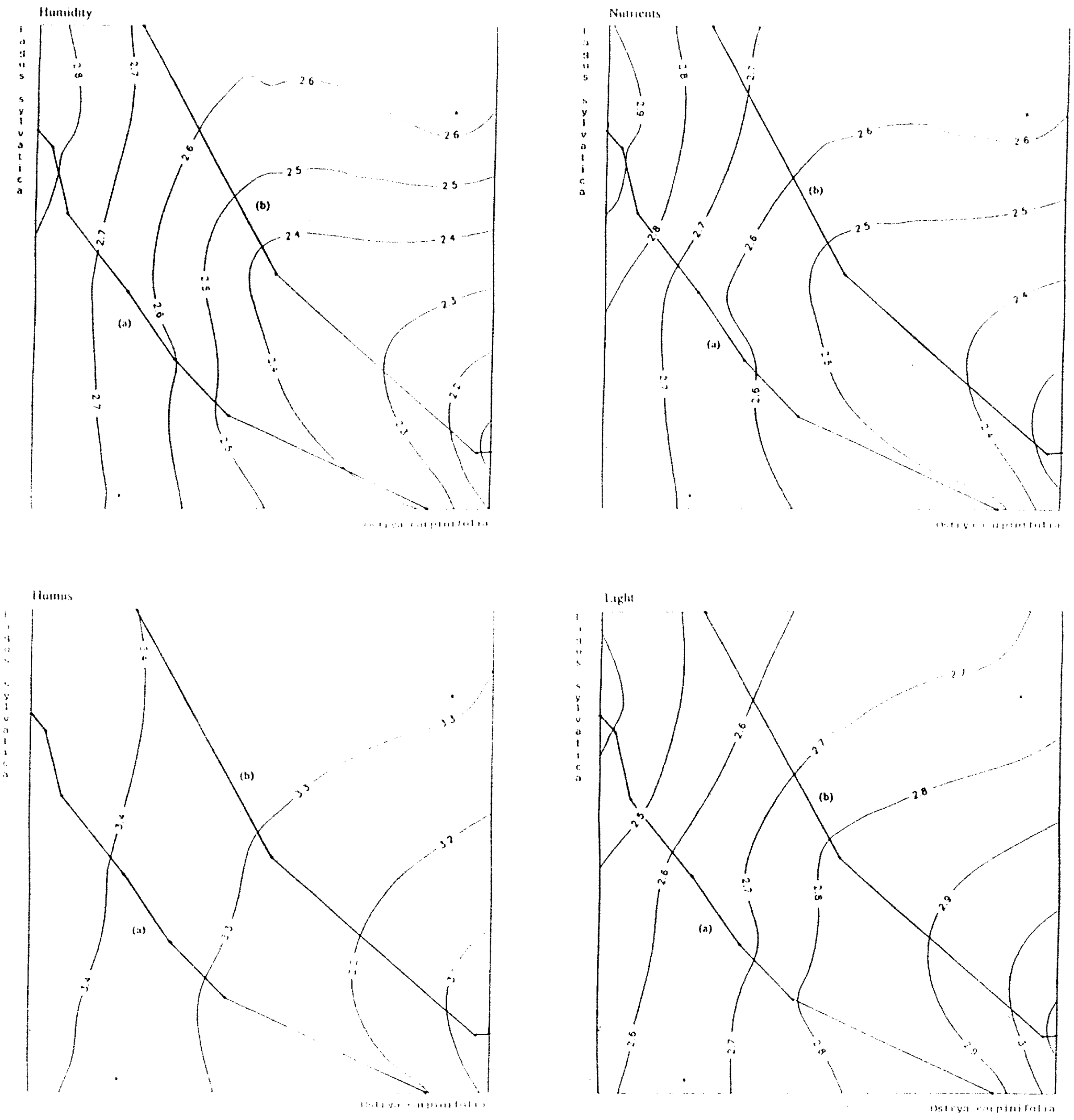

Figure 2. Kriging of humidity, nutrients, humus and light (Table 3) overimposed on ordination given by the *Ostrya* and *Fagus* fuzzy sets (Fig. 1). The trajectories (a) and (b) of Fig. 1 are indicated.

Table 1. Frequency of the main tree species in 13 vegetation types of Poldini (1982). Vegetation types are identified in the caption of Fig. 1.

		VEGETATION TYPES												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1.	Ostrya carpinifolia	5	5	4	2	0	5	2	1	5	5	5	5	5
2.	Fraxinus ornus	5	5	4	5	5	5	1	4	5	3	5	5	5
3.	Quercus pubescens	5	5	1	3	0	1	0	0	0	0	1	4	3
4.	Sorbus aria	4	4	4	2	0	0	1	1	5	5	4	4	4
5.	Quercus petraea	2	4	4	5	5	1	1	1	0	1	0	1	0
6.	Acer campestre	1	5	3	4	2	5	4	4	0	2	0	0	0
7.	Prunus avium	0	3	4	4	3	2	3	1	0	1	0	0	1
8.	Carpinus betulus	1	3	2	3	1	5	4	4	0	1	0	0	0
9.	Tilia cordata	1	2	3	1	0	2	5	2	3	0	1	0	0
10.	Fagus sylvatica	1	2	1	0	0	1	2	2	2	5	2	2	0
11.	Acer pseudoplatanus	0	1	2	2	2	4	5	4	3	3	1	0	0
12.	Fraxinus excelsior	0	1	1	0	0	0	5	5	0	3	0	0	0
13.	Sorbus aucuparia	0	0	2	0	0	0	1	1	0	2	1	1	5
14.	Pinus nigra	0	0	0	0	0	0	0	0	2	1	1	2	5
15.	Pinus sylvestris	0	1	1	0	0	0	0	0	0	0	0	2	4

Table 2. Description of the vegetation types based on frequencies of the species in the growth-life forms. Abbreviations: P = Phanerophytes; NP = Nanophanerophytes; Ch = Chamaephytes; T = Therophytes; G = Geophytes; H = Hemicryptophytes; lian. = lianas; rept. = reptant; frut. = fruticose; suffr. = suffruticose; rad. = radicigemma (root-budding); bulb. = bulbosus; rhiz. = rhizome; ros. = rosulate; scap. = scapose; caesp. = caespitose. Vegetation types are identified in the caption of Fig. 1.

		VEGETATION TYPES												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1.	P caesp.	21.9	22.7	21.3	22.8	10.4	17.2	17.5	16.4	22.2	12.9	19.6	19.3	17.6
2.	P scap.	5.0	9.8	10.3	8.3	10.4	9.6	10.8	11.6	6.3	9.0	7.2	8.7	8.8
3.	P lian.	2.1	3.0	3.3	2.9	3.3	3.6	3.7	2.9	1.9	3.1	2.9	1.4	0.0
4.	Ch rept.	0.4	0.8	0.4	0.4	1.4	2.0	0.7	1.2	0.0	0.4	0.0	0.5	0.0
5.	Ch suffr.	2.9	2.7	1.3	0.0	1.9	0.8	0.0	0.0	1.0	0.4	1.4	9.7	9.8
6.	Ch frut.	0.0	0.0	0.4	0.0	0.5	0.0	0.0	0.0	1.9	0.4	1.4	2.4	2.5
7.	T scap.	0.0	0.8	0.8	1.2	2.4	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.5
8.	G rad.	1.2	1.5	1.3	2.1	0.9	1.2	0.7	0.4	0.0	0.0	2.2	0.0	0.0
9.	G bulb.	3.7	3.0	2.1	5.4	3.3	6.0	4.9	3.3	1.9	2.4	5.1	2.9	3.9
10	G rhiz.	19.4	13.6	13.4	17.8	17.5	17.6	21.3	25.4	16.9	23.9	8.0	9.7	10.3
11	H caesp.	9.9	9.5	10.5	10.0	10.0	7.2	5.6	4.9	8.2	7.5	14.5	10.6	12.7
12	H rept.	0.8	0.8	2.1	1.2	1.9	2.4	2.2	1.2	0.5	1.6	0.0	0.5	1.0
13	H scap.	22.7	22.0	24.3	18.7	28.4	20.4	25.0	23.0	31.4	30.2	29.7	23.2	23.5
14	H ros.	4.1	3.0	3.3	4.6	4.3	6.0	4.5	6.1	1.0	4.3	0.7	2.4	2.5
15	NP	5.8	6.8	4.6	4.6	3.3	6.0	3.0	3.3	6.8	3.9	7.2	7.2	6.9

Table 3. Description of vegetation types on the basis of average ecological indicator values. Vegetation types are identified in the caption of Fig. 1.

	VEGETATION TYPES												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Humidity	2.34	2.47	2.61	2.69	2.64	2.77	2.82	2.92	2.65	2.68	2.40	2.13	2.03
2. pH	3.60	3.52	3.33	3.38	3.10	3.43	3.32	3.37	3.50	3.41	3.64	3.56	3.68
3. Nutrients	2.45	2.54	2.62	2.69	2.62	2.87	2.91	3.01	2.64	2.73	2.53	2.27	2.19
4. Humus	3.28	3.26	3.37	3.38	3.39	3.44	3.46	3.46	3.32	3.40	3.21	3.05	2.99
5. Dispersion	3.32	3.53	3.58	3.61	3.63	3.68	3.66	3.57	3.45	3.49	3.25	3.04	2.83
6. Light	2.83	2.80	2.66	2.65	2.66	2.46	2.41	2.35	2.72	2.53	2.86	3.06	3.17
7. Temperature	3.84	3.78	3.73	3.87	3.77	3.80	3.64	3.54	3.51	3.50	3.58	3.71	3.62
8. Continentality	3.04	2.90	2.84	2.67	2.65	2.60	2.68	2.70	2.94	2.87	3.09	3.19	3.24

Table 4. Ecological description of two trajectories (a) and (b) mapped in Fig. 1. Entries are average ecological values.

	(a)							(b)			
	11	2	3	5	6	7	8	13	12	1	10
Humidity	2.40	2.47	2.61	2.64	2.77	2.82	2.92	2.03	2.13	2.34	2.68
pH	3.64	3.52	3.33	3.10	3.43	3.32	3.37	3.68	3.56	3.60	3.41
Nutrients	2.53	2.54	2.62	2.62	2.87	2.91	3.01	2.19	2.27	2.45	2.73
Humus	3.21	3.26	3.37	3.39	3.44	3.46	3.46	2.99	3.05	3.28	3.40
Dispersion	3.25	3.53	3.58	3.63	3.68	3.66	3.57	2.83	3.04	3.32	3.49
Light	2.86	2.80	2.66	2.66	2.46	2.41	3.35	3.17	3.06	2.83	2.53
Temperature	3.58	3.78	3.73	3.77	3.80	3.64	3.54	3.62	3.71	3.84	3.50
Continentality	3.09	2.90	2.84	2.65	2.60	2.68	2.70	3.24	3.19	3.04	2.87

Table 5. Description of the trajectories (a) and (b) (Fig. 1) according to the raw life-growth form data (Table 2).

	(a)							(b)			
	11	2	3	5	6	7	8	13	12	1	10
P caesp.	20	23	21	10	17	18	16	18	19	22	13
P scap.	7	10	11	10	10	11	12	7	9	5	9
P lian	3	3	3	3	4	4	3	0	1	2	3
Ch rept.	0	1	1	1	2	1	1	0	1	1	1
Ch suffr.	1	3	1	2	1	0	0	10	10	3	1
Ch frut.	1	0	1	1	0	0	0	3	2	0	1
T scap.	0	1	1	2	0	0	0	1	1	0	0
G rad.	2	2	1	1	1	1	1	0	0	1	0
G bulb.	5	3	2	3	6	5	3	4	3	4	2
G rhiz.	8	14	13	18	18	21	25	10	10	19	24
H caesp.	15	10	11	10	7	6	5	13	11	10	8
H rept.	0	1	2	2	2	2	1	1	1	1	2
H scap.	30	22	24	28	20	25	23	24	23	23	30
H ros.	1	3	3	4	6	5	6	3	2	4	4
NP	7	7	5	3	6	3	3	7	7	6	4

Table 6. Degrees of belonging (x100) of vegetation types to the fuzzy sets defined by life-growth forms and leading species (*Ostrya carpinifolia*, *Quercus petraea* and *Fagus sylvatica*). The table is structured according to the two trajectories (a) and (b) (Fig. 1).

	(a)							(b)			
	11	2	3	5	6	7	8	13	12	1	10
P caesp.	71	100	88	59	50	19	20	55	72	95	0
P scap.	5	35	70	95	98	99	100	0	3	36	86
P lian.	11	58	88	98	100	93	95	0	9	51	71
Ch rept.	8	48	81	97	100	96	97	0	6	44	78
Ch suffr.	99	59	36	36	25	3	0	99	100	84	14
Ch frut.	95	36	16	31	21	5	0	100	95	71	23
T scap.	71	100	89	59	40	20	20	54	71	95	0
G rad.	51	99	95	60	53	26	28	32	52	85	0
G bulb.	84	10	0	36	25	11	4	92	83	58	34
G rhiz.	60	40	75	96	100	99	99	0	3	39	84
H caesp.	99	82	59	45	33	2	0	92	100	99	1
H rept.	9	52	83	98	100	95	96	0	7	46	76
H scap.	54	0	21	83	80	81	75	65	51	44	100
H ros.	9	51	84	98	100	95	96	0	7	46	76
NP	99	57	34	35	24	4	0	100	99	83	15
Ostrya	86	43	31	20	6	3	0	100	97	53	23
Quercus	49	91	92	88	77	62	55	0	25	80	38
Fagus	0	19	31	45	61	75	78	12	12	49	100

Table 7. Fuzziness of the life-growth form fuzzy sets in the two trajectories (a) and (b) (Fig. 1).

		(a)			(b)		
	Fuzzy set	Relative Entropy	F1	F2	Relative Entropy	F1	F2
1.	P caesp.	0.685	0.489	0.399	0.534	0.390	0.290
2.	P scap.	0.373	0.223	0.176	0.430	0.265	0.213
3.	P lian	0.401	0.226	0.182	0.576	0.435	0.320
4.	Ch rept.	0.391	0.243	0.180	0.519	0.360	0.275
5.	Ch suffr.	0.564	0.406	0.325	0.325	0.155	0.143
6.	Ch frut.	0.541	0.326	0.279	0.483	0.285	0.245
7.	T scap.	0.680	0.460	0.393	0.538	0.400	0.294
8.	G rad.	0.717	0.560	0.435	0.628	0.475	0.363
9.	G bulb	0.514	0.291	0.255	0.742	0.505	0.437
10.	G rhiz	0.451	0.317	0.235	0.448	0.290	0.226
11.	H caesp	0.541	0.400	0.298	0.141	0.050	0.048
12.	H rept.	0.395	0.243	0.182	0.539	0.385	0.290
13.	H scap.	0.661	0.423	0.371	0.751	0.645	0.484
14.	H ros.	0.392	0.243	0.179	0.539	0.385	0.290
15.	NP	0.566	0.403	0.315	0.337	0.165	0.151

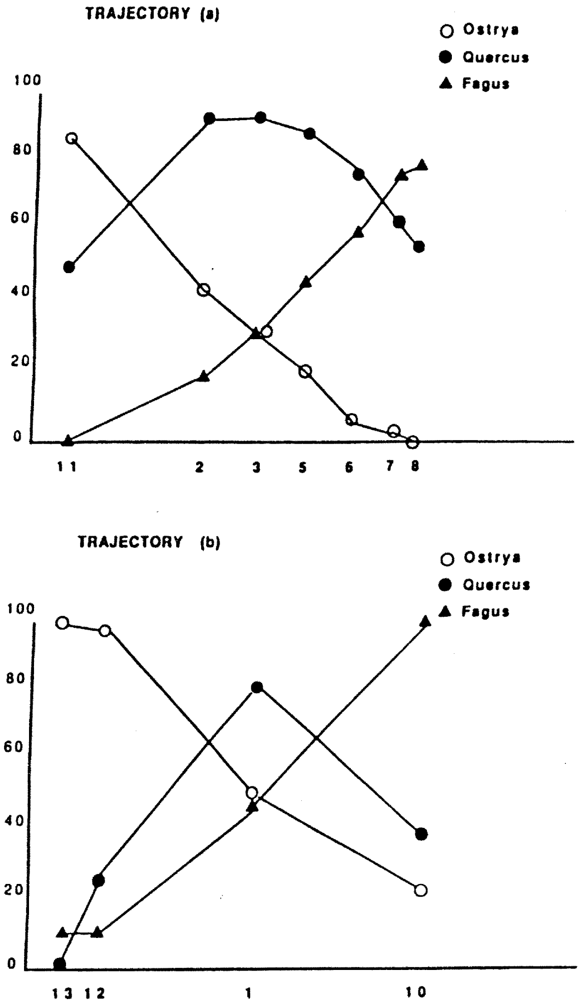

Figure 3. Degrees of belonging (x100) of vegetation types to the fuzzy sets of *Ostrya carpinifolia, Quercus petraea* and *Fagus sylvatica* represented along the two trajectories (a) and (b) (Fig. 1).

sets of life-growth forms for the two trajectories is presented in Table 7. This table shows that there are life-growth forms with low fuzziness (less than 0.50 of De Luca-Termini's relative entropy or less than 0.30 in at least one of the Yager's fuzziness measures) in both trajectories, but less in one, (a) or (b), and that there are life-growth forms with low fuzziness only in one of the two trajectories.

Discussion and conclusions

Trends fitting theoretical models of coenoclines are detected by using deviations from expectation (Orlóci 1981, Feoli and Orlóci 1985). In the present study, theoretical models emerge from another transformation of the data. In both, the original data are *manipulated* to reduce random variation. However, in the first case the manipulation is based on internal criteria, as normally is done in spectral analysis and time series analysis (*e.g.* in climatology), while in the second case the manipulation is based on external assumptions underlying a given classification.

In the present study life-growth forms are used in a structural description of vegetation types. The types, defined on a floristic basis (Poldini 1982), were collated into three main groups by Feoli, Ganis and Poldini (1987) based on several other characters. These three groups have been interpreted as three main stages in succession, considering the frequency of *Ostrya*, *Quercus* and *Fagus*. These are used to impose a structural order on the fuzzy sets defined by the trees. The ordination given by the *Ostrya* and *Fagus* fuzzy sets reveals two possible successional trajectories. The trajectories cannot be linked directly with time, but the distances between the successive stages may suggest the "fluidity" of the transitions. According to each trajectory, the coenocline model proves to be acceptable for explaining structural vegetation changes during succession. Clearly, fuzzy set theory provides tools that can improve the classification approach in vegetation study and scrutinize the data collected for syntaxonomical purposes for evidence on vegetation dynamics. External classifications are not only useful to reduce random variation via an averaging process, but also to test if the imposed classification is acceptable in terms of some underlying model (in this case coenocline in succession). Classifying means to introduce into data set logical criteria for conceptualizing the classes. Through this new information is revealed about the relationships of the elements of the classification. Thus, Syntaxonomy, the end result of a classification process, can be the source of useful information in ecological analysis. Syntaxonomy coupled with fuzzy set theory help to test models and to verify predictions from data that have been collected only for descriptive purposes as shown by the present paper.

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Appendix

Fuzziness measures the indeterminateness of a fuzzy set. According to De Luca and Termini (1972), a measure of fuzziness, d(A), must have the following properties. Let $u_A(x)$ be the membership function of fuzzy set A for $x \in X$, X finite, cA the complement of the fuzzy set A with elements $u_{cA}(x) = 1-u_A(x)$, and A' another fuzzy set with the same elements:

1) d(A) = 0 if A is a crisp set in X;

2) d(A) assumes a unique maximum if $u_A(x) = 0.5$ with $x \in X$;

3) d(A)> d(A') if A' is crisper than A;

4) d(cA) = d(A).

De Luca and Termini (1972) suggest the "entropy" of a fuzzy set $\mathbf{A} = (x, u_A(x))$ as a measure of fuzziness:

$$d(\mathbf{A}) = H(\mathbf{A}) + H(c\mathbf{A}),\ x \in X$$

where

$$H(A) = -K \sum_{i=1}^{n} u_A(x_i) \ln u_A(x_i)$$

$$H(cA) = -K \sum_{i=1}^{n} (1 - u_A(x_i)) \ln (1 - u_A(x_i))$$

where n is the number of elements and K a positive constant (in our case $K=1$). On the basis of property 2, the relative entropy, rd(**A**), is defined as:

$$rd(\mathbf{A}) = d(\mathbf{A})/d(\mathbf{A})_{max}$$

where $d(\mathbf{A})_{max} = n \ln 2$. Since the relative entropy satisfies properties 1-4, it is a relative measure of fuzziness.

Yager (1979) proposes to measure the fuzziness on the basis of the distance of the fuzzy set from its complement. This measure also satisfies properties 1-4. The distance is defined by

$$Dp(A,cA) = \left(\sum_{i=1}^{n} | u_A(x_i) - u_{cA}(x_i) |^{p} \right)^{1/p}$$

where p is a positive integer. A measure of fuzziness of fuzzy set **A** is:

$$Fp(A) = 1 - (Dp(A,cA) / n^{1/p}).$$

In this paper we have used $p=1$ and $p=2$. On the basis of Yager's measure, a similarity between two fuzzy sets **A** and **A'** can be expressed as

$$Sp(A,A') = 1 - (Dp(A,A') / n^{1/p}).$$

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