COENOSES 9(1):25-32 (1994)

NAIVETÉ OF FUZZY SYSTEM SPACES IN VEGETATION DYNAMICS ?

E. Feoli[1,2] and V. Zuccarello[2]

1 Department of Biology, University of Trieste, Italy
2 International Center for Theoretical and Applied Ecology, Gorizia, Italy

Keywords: Community dynamics, Fuzzy sets, Ordinations.

Abstract. The application of fuzzy set theory for describing vegetation dynamics is discussed on the basis of results obtained with forest data of northeastern Italy.

Introduction

Vegetation is a complex system whose state variables and the relationships between them are difficult to understand and/or to measure. The notion of complexity has many definitions (Flood & Carson 1988, Orlóci 1991a) and as was illustrated by Klir & Folger (1988) it is directly related to the interest and capabilities of the investigator:

"*to the neurologist the brain, as a feltwork of fibers and a soup of enzymes, is certainly complex; and equally the transmission of a detailed description of it would require much time. To the butcher the brain is simple, he has to distinguish it from only about thirty other "meats" (Ashby 1973)*".

For this reason, to avoid philosophical discussions, we would prefer to associate the idea of complexity to a concept which has a clear definition such as the complex number. A complex number ζ is the sum of a real number a and an imaginary part ib:

$\zeta = a + ib$

where b is another real number and i is the imaginary quantity.

As a consequence of this we define complex the system in which at least one state variable or one relationship between two state variables is unmeasurable by any available instrumentation. In this way the idea of complexity is strictly related to the impossibility of the observer to give a direct metric representation of the system under study.

Any number derived from a measurement made on the nominal or ordinal scale may be considered as a complex number. In it the actual value a is kept hidden by the imaginary part ib. The latter is a quantity without a meaningful numerical expression where i is interpretable as an imaginary operator used by the researcher according to his personal experience. In analogy with the familiar $(-1)^{1/2}$, ib may be considered as a catalyst allowing to account for the "unmeasurable" and to proceed with the research in some logical and quantitative way (see Nijkamp, Leitner & Wrigley 1985).

However, while the use of $(-1)^{1/2}$ could lead to precise predictions, the use of nominal and/or ordinal scales condemns the prediction to a state of imprecision and uncertainty. We may ask if this imprecison would be unacceptable in vegetation study. The answer to this question may be impossible, and the question could even be irrelevant. The meaning of precision in the context of what it should be relevant to depict when we are dealing with complex systems would require deep philosophical discussions that would lead far from the aim of this paper. In this respect we should answer a more fundamental questions. Namely, what we want or what we could expect from an analysis of a complex system? If we define what we want, is it relevant or useful for the advancement of Science? Is the precision we are able to work with acceptable in this context? There are two kinds of relevance to be considered: the relevance of a component of the system (state or control variables) and the relevance of the precision by which the component should be measured. A component is highly relevant if its impact on the system is highly consistent with the behaviuor of the system. A relevant component should be measured with high precision only if its small variations influence in a big way all variation of the system.

Vegetation may be seen under different perspectives according to the interest of the researchers, consequently its states may be described by different sets of attributes (Feoli 1984). Each set of variables defines a multidimensional space representing more or less efficiently the ecological space (Feoli & Orlóci 1991). Since vegetation changes in time, vegetation trajectories can be described in the ecological space (Roberts 1987,1989 a,b; Feoli & Zuccarello 1992). This space is usually represented by p combinations (ordination axes) of the m measured environmental variables ($p < m$). There are many ways to represent the ecological space by linear or non- linear models. In the descriptive approach (Goodall 1970) these models are used to infer the vegetation variation in time or space. However, as suggested by Bezdek (1987) when the system uncertainty is large as a consequence of its complexity, fuzzy inference should be used instead of probabilistic inference (see also Zadeh 1973, Rosen 1979, Bosserman & Ragade 1982). This approach fits well with the complex nature of vegetation that forces us to use ill-defined

Table 1. Frequency of the leading tree species in 13 vegetation types of Poldini (1982). Vegetation types code: 1 = *Mercuriali ovatae-Ostryetum*; 2 = *Buglossoido-Ostryetum capinifoliae polygaletosum;* 3 = *Buglossoido-Ostryetum capinifoliae hieracetosum*; 4 = *Carici-Quercetum petraeae violetosum;* 5 = *Carici-Quercetum petraeae quercetosum*; 6 = *Ornithogalo-Carpinetum betuli;* 7 = *Carpino-Fraxinetum excelsioris tilietosum*; 8 = *Carpino-Fraxinetum excelsioris cerastietosum*; 9 = *Hemerocallido-Ostryetum carpinifoliae;* 10 = *Ostryo-Fagetum*; 11 = Ass. *Betonica alopecuros-Ostrya carpinifolia;* 12 = *Orno-Pinetum ostryetosum*; 13 = *Orno-Pinetum nigrae pinetosum nigrae.*

		Vegetation types												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1.	Ostrya carpinifolia	5	5	4	2	0	5	2	1	5	5	5	5	5
2.	Fraxinus ornus	5	5	4	5	5	5	1	4	5	3	5	5	5
3.	Quercus pubescens	5	5	1	3	0	1	0	0	0	0	1	4	3
4.	Sorbus aria	4	4	4	2	0	0	1	1	5	5	4	4	4
5.	Quercus petraea	2	4	4	5	5	1	1	1	0	1	0	1	0
6.	Acer campestre	1	5	3	4	2	5	4	4	0	2	0	0	0
7.	Prunus avium	0	3	4	4	3	2	3	1	0	1	0	0	1
8.	Carpinus betulus	1	3	2	3	1	5	4	4	0	1	0	0	0
9.	Tilia cordata	1	2	3	1	0	2	5	2	3	0	1	0	0
10.	Fagus sylvatica	1	2	1	0	0	1	2	2	2	5	2	2	0
11.	Acer pseudoplatanus	0	1	2	2	2	4	5	4	3	3	1	0	0
12.	Fraxinus excelsior	0	1	1	0	0	0	5	5	0	3	0	0	0
13.	Sorbus aucuparia	0	0	2	0	0	0	1	1	0	2	1	1	5
14.	Pinus nigra	0	0	0	0	0	0	0	0	2	1	1	2	5
15.	Pinus sylvestris	0	1	1	0	0	0	0	0	0	0	0	3	4

sampling designs and complex or "vague" (nominal or ordinal scale) descriptions of the units (relevés) (De Patta Pillar & Orlóci 1993). Since in this context a reasonable evaluation of the random components (error and variability) is all but impossible, it is obviuos that classical statistics (Fisherian statistics, see Orlóci 1991 a,b) is inadequate both for description and inference.

In fuzzy set theory the fuzzy set is defined by the elements it includes and by the degrees of belonging (or membership) of the elements to the set itself (Zadeh 1965). The degree of belonging is a real number ranging between zero and one. In vegetation studies the fuzzy sets may be defined by environmental and/or biological variables and by vegetation types that describe the vegetation system under a study. The fuzzy sets may be used to define a fuzzy system space in which they are the reference axes (Roberts 1986, Feoli & Zuccarello 1986, 1988, 1992, Dale 1988, Marsili-Libelli 1989, Feoli and Orlóci 1991). In spite of the fact that the state variables may be measured on nominal and/or ordinal scales the fuzzy sets represent always a continuous multidimensional space. This makes fuzzy set theory an appealing tool to represent in a simple way the complexity of vegetation system. In this paper we show how the description of the vegetation system by fuzzy sets can be used to infer about vegetation dynamics. We do not want to enter into the philosophical discussion on the data; we accept what we have as reasonable to describe the vegetation in an abstract way under the Braun-Blanquet approach (Westoff & van der Maarel 1973). We are convinced that this approach is very useful to understand and to conceptualize vegetation structure and dynamics.

The data

The same data set that Feoli & Zuccarello (1992) used is used in this paper. This set consists of three data matrices that describe 13 vegetation types following Poldini (1982) and Feoli, Ganis and Poldini (1987):

1) Matrix **T** (15x13) - the frequency classes for the leading tree species in the vegetation types (Table 1).

2) Matrix **L** (15x13) - the relative frequencies of the Mueller-Dombois and Ellenberg (1974) life-growth forms in the vegetation types (Table 2).

3) Matrix **E** (8x13) - the Landolt's (1977) average indicator values of the vegetation types (Table 3).

Matrices **L** and **E** were constructed on the basis of the constancy of all the species in the synthetic tables of Poldini (1982). This type of data has been discussed by Feoli & Scimone (1985).

Methods

Given a data matrix description of a set of vegetation states, the ecological space may be represented by two groups of ordination methods. One uses only the intrinsic information (the information in the data matrix), such as traditional ordination methods (PCA, CA, CCA, DCA, NMDS, etc.). The second one uses extrinsic information (information not included in the data matrix) that can be infused in the analysis in different ways. In this group the ordination axes are conditioned by their ecological meaning determined *a posteriori,* as in discriminant analysis (Cooley & Loohnes 1965, Gittins 1985) and concentration analysis (Feoli & Orlóci 1979), or a priori, as in fuzzy set ordination (Feoli &

Table 2. Description of the vegetation types based on frequencies of the species in the growth-life forms. Abbreviations: P = Phanerophytes; NP = Nanophanerophytes; Ch = Chamaephytes; T = Therophytes; G = Geophytes; H = Hemicryptophytes; lian. = lianas; rept. = reptant; frut. = fruticose; suffr. = suffruticose; rad. = radicigemma (root-budding); bulb. = bulbosus; rhiz. = rhizome; ros. = rosulate; scap. = scapose;caesp. = caespitose. Vegetation types are identified in the caption of Table 1.

		Vegetation types												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1.	P caesp.	21.9	22.7	21.3	22.8	10.4	17.2	17.5	16.4	22.2	12.9	19.6	19.3	17.6
2.	P scap.	5.0	9.8	10.3	8.3	10.4	9.6	10.8	11.6	6.3	9.0	7.2	8.7	8.8
3.	P lian.	2.1	3.0	3.3	2.9	3.3	3.6	3.7	2.9	1.9	3.1	2.9	1.4	0.0
4.	Ch rept.	0.4	0.8	0.4	0.4	1.4	2.0	0.7	1.2	0.0	0.4	0.0	0.5	0.0
5.	Ch suffr.	2.9	2.7	1.3	0.0	1.9	0.8	0.0	0.0	1.0	0.4	1.4	9.7	9.8
6.	Ch frut.	0.0	0.0	0.4	0.0	0.5	0.0	0.0	0.0	1.9	0.4	1.4	2.4	2.5
7.	T scap.	0.0	0.8	0.8	1.2	2.4	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.5
8.	G rad.	1.2	1.5	1.3	2.1	0.9	1.2	0.7	0.4	0.0	0.0	2.2	0.0	0.0
9.	G bulb.	3.7	3.0	2.1	5.4	3.3	6.0	4.9	3.3	1.9	2.4	5.1	2.9	3.9
10	G rhiz.	19.4	13.6	13.4	17.8	17.5	17.6	21.3	25.4	16.9	23.9	8.0	9.7	10.3
11	H caesp.	9.9	9.5	10.5	10.0	10.0	7.2	5.6	4.9	8.2	7.5	14.5	10.6	12.7
12	H rept.	0.8	0.8	2.1	1.2	1.9	2.4	2.2	1.2	0.5	1.6	0.0	0.5	1.0
13	H scap.	22.7	22.0	24.3	18.7	28.4	20.4	25.0	23.0	31.4	30.2	29.7	23.2	23.5
14	H ros.	4.1	3.0	3.3	4.6	4.3	6.0	4.5	6.1	1.0	4.3	0.7	2.4	2.5
15	NP	5.8	6.8	4.6	4.6	3.3	6.0	3.0	3.3	6.8	3.9	7.2	7.2	6.9

Table 3. Description of vegetation types on the basis of average ecological indicator values. Vegetation types are identified in the caption of Table 1.

		Vegetation types												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1.	Humidity	2.34	2.47	2.61	2.69	2.64	2.77	2.82	2.92	2.65	2.68	2.40	2.13	2.03
2.	pH	3.60	3.52	3.33	3.38	3.10	3.43	3.32	3.37	3.50	3.41	3.64	3.56	3.68
3.	Nutrients	2.45	2.54	2.62	2.69	2.62	2.87	2.91	3.01	2.64	2.73	2.53	2.27	2.19
4.	Humus	3.28	3.26	3.37	3.38	3.39	3.44	3.46	3.46	3.32	3.40	3.21	3.05	2.99
5.	Dispersion	3.32	3.53	3.58	3.61	3.63	3.68	3.66	3.57	3.45	3.49	3.25	3.04	2.83
6.	Light	2.83	2.80	2.66	2.65	2.66	2.46	2.41	2.35	2.72	2.53	2.86	3.06	3.17
7.	Temperature	3.84	3.78	3.73	3.87	3.77	3.80	3.64	3.54	3.51	3.50	3.58	3.71	3.62
8.	Continentality	3.04	2.90	2.84	2.67	2.65	2.60	2.68	2.70	2.94	2.87	3.09	3.19	3.24

Zuccarello 1986, 1988, Roberts 1986). In the fuzzy case each axis represents the degree of belonging of a set of elements to a given fuzzy set. It is interpretable a priori because the fuzzy set is defined according to external assumptions related to the causal hypothesis underlying its existence.

In the present paper, fuzzy sets are obtained on the basis of the external assumptions that defines three dynamical stages of the well-known successional sequence of the broad leaved mixed forests in North East Italy (Feoli & Zuccarello 1992): the initial stage characterized by *Ostrya carpinifolia*, the intermediate stage characterized by *Quercus pubescens* or *Quercus petraea* and the final stage characterized by *Fagus sylvatica.* On the basis of successional stages the vegetation types (see Table 1) have been grouped into three sets. The centroids of these sets have been used in a process of subsequent matrix multiplications giving rise to new matrices of fuzzy sets:

1) C(L) **L** = **F**(V;S)

2) **F**(V;S) **L'** = **F**(L;S)

3) C(L) **F**(V;S) = **F**(V;L)

4) C(T) **F**(V;S) = **F**(V;T)

5) C(T) **F**(L;S) = **F**(L;T)

6) **F**(L;T)' **F**(L;T) = **F**(T;T)

7) **F**(V;T) **E'** = **F**(E;T)

In these **C** signifies "centroid". The others symbols are as follows:

1) **F**(V;S) is the "degree of belonging" matrix of the 13 vegetation types (V) to the three vegetation stages (S) of the succession;

2) **F**(L;S) is the "degree of belonging" matrix of the life-growth forms (L) to the three vegetation stages;

3) **F**(V;L) is the "degree of belonging" matrix of the vegetation types to the sets defined by life-growth forms. The

degree of belonging is conditioned by the three main stages of succession. This matrix represents the links between the life-growth forms and the vegetation types; and so on.

The degrees of belonging in the product matrices have been obtained by normalizing the columns and by transforming the values of the rows between 0 and 1. Other matrices of fuzzy sets may be calculated:

8) **C**(E) **F**(V;S) = **F**(V;E)

9) **C**(E) **F**(L;S) = **F**(L;E)

and so on by using the centroids of matrix **E**. However we concentrate our analysis on matrices **F**(V;T), **F**(L;T), **F**(T;T), **F**(E;T). These matrices show the degree of belonging of vegetation types, life-growth forms, trees and ecological indices to the fuzzy sets defined by the trees (tree fuzzy sets). These fuzzy sets define a fuzzy system space, conditioned by the three main stages of succession, where the position of objects (vegetation types, life-growth forms, trees and ecological indices) is determined by the degrees of belonging to the tree fuzzy sets. The comparison between the tree fuzzy sets has been done by applying the similarity based on Yager's (1979) measure of fuzziness, as suggested by Feoli and Zuccarello (1992). Eigenanalysis has been applied to the similarity matrix to get the ordination of the tree fuzzy sets by using the second and third eigenvectors (Feoli 1977). This analysis has been done to obtain an ordered sequence of trees. This sequence has a dynamical meaning because it is conditioned by the three stages of succession. The degrees of belonging of vegetation types, life-growth forms, trees and ecological indices to the tree fuzzy sets is used for a fuzzy inference on the dynamical pattern of vegetation variation.

Results

The ordinations of the fuzzy sets in **F**(V;T), **F**(L;T), **F**(T;T), **F**(E;T) given by the second and third eigenvectors of Yager's similarity matrices arrange the trees along a curved line in the following sequence: *Pinus nigra, Sorbus aucuparia, Ostrya carpinifolia, Pinus sylvestris, Sorbus aria, Fraxinus ornus, Quercus pubescens, Quercus petraea, Prunus avium, Acer campestre, Carpinus betulus, Tilia cordata, Acer pseudoplatanus, Fraxinus excelsior* and *Fagus sylvatica.* Table 4 presents the degree of belonging of vegetation types to the sets defined by the trees (they are ordered according to the ordination obtained by the second and third eigenvectors of Yager's similarity matrices). Fig. 1 (a,b) shows the pattern of the degrees of belonging of vegetation types along the trees sequence. The types are grouped according to the two trajectories defined by Feoli & Zuccarello (1992). These trajectories correspond to different successional sequences each related to specific edaphic conditions. The types (4 and 9) outside of the trajectories are not considered in the figure.

Table 5 presents the degree of belonging of life-growth forms to the sets defined by trees. Table 6 shows the degree of belonging of the trees to sets defined by themselves. Table 7 shows the degree of belonging of ecological indicator values to the tree fuzzy sets.

Discussion

Vegetation Classification and fuzzy sets

Classification is a process by which sets of objects are defined on the basis of different criteria. Once the sets are established, to find degrees of belonging of objects to the sets is a natural consequence. This was mathematically codified

Table 4. Degree of belonging (x 100) of the vegetation types to the sets defined by the trees (tree fuzzy sets). The order of the fuzzy sets follows a dynamic sequence described in the text. Vegetation types are identified in the caption of Table 1.

		Vegetation types												
Nr.	Tree fuzzy sets	1	2	3	4	5	6	7	8	9	10	11	12	13
14	Pinus n.	53	41	29	16	19	6	3	0	94	24	85	97	100
13	Sorbus au.	52	42	30	18	20	7	3	0	93	23	86	97	100
1	Ostrya c.	52	43	31	19	20	7	3	0	91	23	86	97	100
15	Pinus s.	60	56	43	36	30	13	5	0	83	17	95	100	99
4	Sorbus ar.	61	57	44	37	31	13	5	0	83	17	95	100	98
2	Fraxinus o.	75	75	62	58	46	23	8	0	68	10	100	95	83
3	Quercus pu.	84	89	76	76	58	31	11	0	52	4	100	86	66
5	Quercus pe.	80	91	92	100	88	77	62	56	8	38	49	25	0
7	Prunus a.	87	92	97	100	98	92	82	77	22	62	45	23	0
6	Acer c.	84	82	91	87	97	100	97	95	38	85	33	18	0
8	Carpinus b.	80	75	85	79	94	100	100	99	43	92	26	15	0
9	Tilia c.	74	65	77	69	87	97	100	100	47	96	20	12	0
11	Acer ps.	66	54	67	56	79	92	98	100	51	99	12	8	0
12	Fraxinus e.	58	44	58	45	71	86	95	98	53	100	5	5	0
10	Fagus s.	49	19	31	3	45	61	75	78	82	100	0	12	12

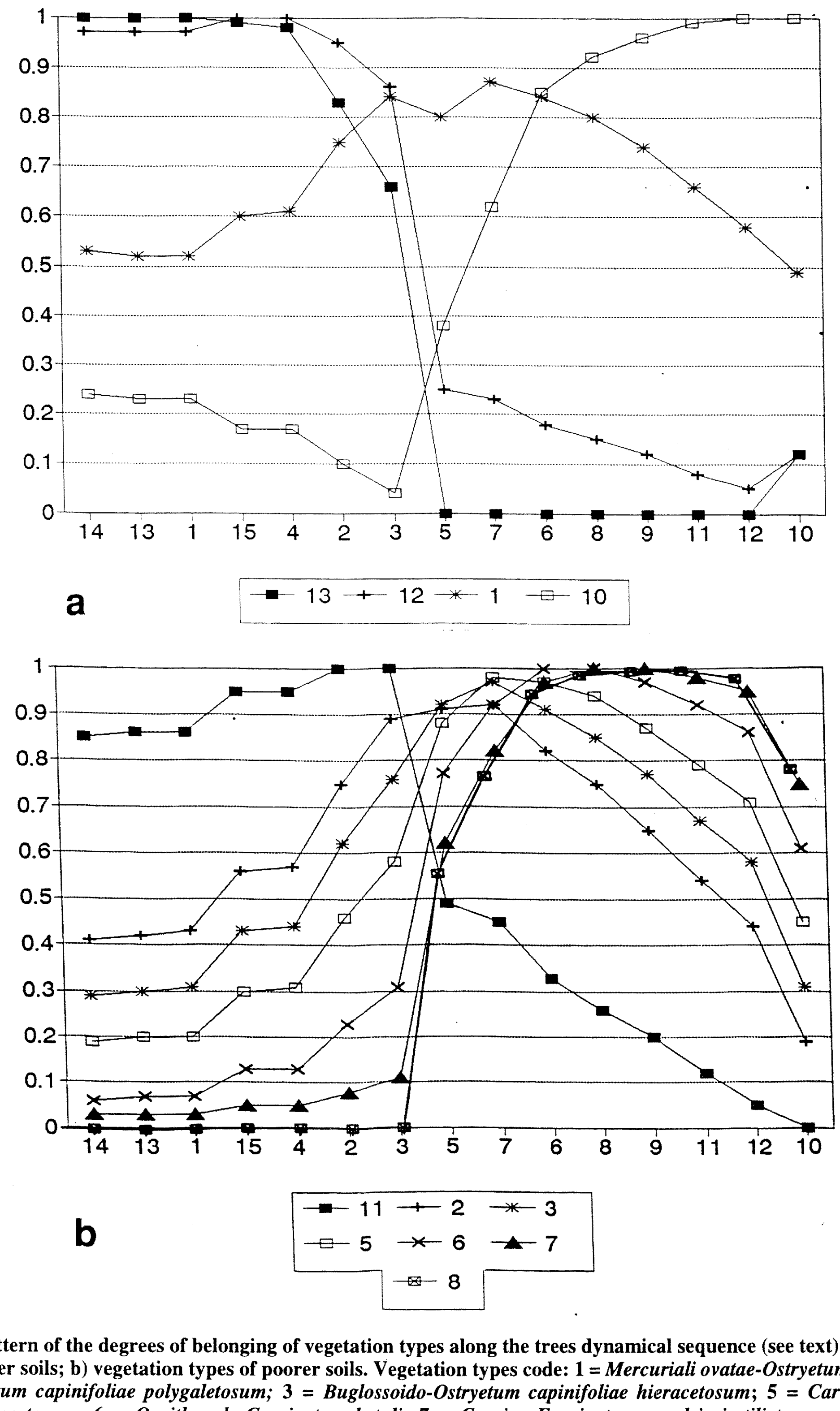

Figure 1. Pattern of the degrees of belonging of vegetation types along the trees dynamical sequence (see text). a) vegetation types of richer soils; b) vegetation types of poorer soils. Vegetation types code: 1 = *Mercuriali ovatae-Ostryetum;* 2 = *Buglossoido-Ostryetum capinifoliae polygaletosum;* 3 = *Buglossoido-Ostryetum capinifoliae hieracetosum*; 5 = *Carici-Quercetum petraeae quercetosum;* 6 = *Ornithogalo-Carpinetum betuli;* 7 = *Carpino-Fraxinetum excelsioris tilietosum*; 8 = *Carpino-Fraxinetum excelsioris cerastietosum*; 10 = *Ostryo-Fagetum;* 11 = Ass. *Betonica alopecuros-Ostrya carpinifolia;* 12 = *Orno-Pinetum ostryetosum*; 13 = *Orno-Pinetum nigrae pinetosum nigrae.*

Table 5. Degree of belonging (x 100) of the life-growth forms to the sets defined by the main trees (tree fuzzy sets). The order of the fuzzy sets follows the dynamical sequence described in the text.
U=humidity, R=soil reaction, N=nutrients, Hu=humus, D=soil texture, L=light, T=temperature, K=continentality

		Tree fuzzy sets														
		14	13	1	15	4	2	3	5	7	6	8	9	11	12	10
1	P caesp.	79	80	79	92	94	100	98	54	36	20	14	8	5	3	5
2	P scap.	0	0	0	0	0	4	7	71	83	99	100	99	96	93	84
3	P lian	7	7	7	9	10	20	23	78	88	100	99	96	91	88	78
4	Ch rept.	5	5	5	6	6	13	16	74	86	100	100	98	94	91	82
5	Ch suffr.	97	97	97	100	100	85	77	22	5	0	0	0	3	7	24
6	Ch frut.	100	100	100	99	99	78	68	14	0	0	3	5	10	14	36
7	T scap.	48	49	48	68	70	95	100	86	73	52	40	30	21	14	0
8	G rad.	28	29	28	46	49	78	84	97	91	77	67	57	46	39	21
9	G bulb	2	3	2	18	20	48	56	100	100	92	83	74	63	55	30
10	G rhiz	12	11	12	6	5	0	0	57	72	94	99	100	100	100	100
11	H caesp	87	87	87	97	98	96	92	41	23	9	5	1	0	0	7
12	H rept.	6	5	6	5	5	8	10	69	82	99	100	99	96	94	87
13	H scap.	91	90	91	75	73	38	26	0	1	25	35	43	53	51	95
14	H ros.	5	4	5	4	4	8	11	71	83	99	100	99	96	93	86
15	NP	95	95	95	99	100	89	82	28	11	4	3	2	4	6	20

Table 6. Degree of belonging (x 100) of the main tree species to the sets defined by the trees (tree fuzzy sets). The order of the fuzzy sets follows the dynamical sequence described in the text.

		Tree fuzzy sets														
Nr.	Main trees	14	13	1	15	4	2	3	5	7	6	8	9	11	12	10
14	Pinus n.	100	100	100	99	99	95	87	28	9	2	1	0	3	5	34
13	Sorbus au.	100	100	100	99	99	95	88	29	10	2	1	0	3	5	33
1	Ostrya c.	100	100	100	99	99	95	88	29	10	3	1	0	3	5	33
15	Pinus s.	99	99	99	100	100	98	93	35	15	5	2	0	2	3	28
4	Sorbus ar.	99	99	99	100	100	98	94	36	16	5	2	0	2	3	28
2	Fraxinus o.	94	95	95	98	98	100	98	44	22	8	4	0	0	0	20
3	Quercus pu.	86	86	87	93	93	98	100	54	31	14	8	3	1	0	14
5	Quercus pe.	0	1	2	12	12	26	43	100	96	87	81	75	70	66	46
7	Prunus a.	0	1	1	8	9	19	33	97	100	97	94	90	87	83	63
6	Acer c.	0	0	0	4	5	11	22	90	97	100	100	98	97	95	79
8	Carpinus b.	0	0	0	3	3	8	18	86	94	100	100	100	99	97	83
9	Tilia c.	0	0	0	2	2	5	14	82	91	99	100	100	100	99	86
11	Acer ps.	0	0	0	1	1	3	10	78	88	97	99	100	100	100	90
12	Fraxinus e.	0	0	0	0	0	1	7	74	84	95	97	99	100	100	92
10	Fagus s.	14	13	12	8	8	1	0	49	56	73	78	82	87	90	100

by Zadeh (1965), however, the approach was always implicit in common practice, in some scientific disciplines, it was very evident in phytociology. For example, Braun-Blanquet (see Westhoff & Maarel 1973) has distinguished five levels of "fidelity" of a species in different syntaxa (5 = exclusive; 4 = selective; 3 = preferential; 2 = indifferent; 1 = accidental) which is the same concept as the "degree of belonging". If a species is exclusive of a vegetation type then it has the highest degree of belonging to that type, and the type has the highest degree of belonging to the set defined by that species.

The definition of different vegetation types involves the process of classification and typification (Braun-Blanquet 1964). A vegetation type represents a set of vegetation states with specific common characteristics. Vegetation types may be defined at different hierarchical levels. In syntaxonomy there are 4 main levels: association, alliance, order and class. As was shown by Feoli and Zuccarello (1986, 1988), for each vegetation type a corresponding fuzzy set may be obtained. This gives degrees of belonging of relevés or sets of relevés (centroids, synthetic tables, etc. of syntaxa) and species or other characters to the set defined by the vegetation types.

All the syntaxa are abstract entities, notwithstanding that each association table contains relevés that are descriptions of concrete pieces of land under a specific kind of vegetation. There are at least two reasons for considering the syntaxa as abstract entities: 1) a syntaxon supposed to include all pieces

Table 7. Degree of belonging (x 100) of the ecological indicator values to the sets defined by the main trees (tree fuzzy sets). The order of the fuzzy sets follows the dynamical sequence described in the text.

Nr.	Tree fuzzy sets	U	R	N	Hu	D	L	T	K
14	Pinus n.	0	75	7	31	15	100	50	94
13	Sorbus au.	0	75	7	31	15	100	50	94
1	Ostrya c.	0	75	8	30	15	100	50	94
15	Pinus s.	0	73	6	31	18	100	54	91
4	Sorbus ar.	0	74	7	32	18	100	54	92
2	Fraxinus o.	0	72	5	33	23	100	60	88
3	Quercus pu.	0	69	4	35	29	100	68	82
5	Quercus pe.	89	27	80	76	100	7	78	0
7	Prunus a.	98	27	88	78	100	0	69	0
6	Acer c.	100	30	92	78	94	0	59	8
8	Carpinus b.	100	32	93	78	91	0	58	10
9	Tilia c.	100	32	93	76	89	0	55	12
11	Acer ps.	100	32	94	76	88	0	51	13
12	Fraxinus e.	100	33	94	75	85	0	49	14
10	Fagus s.	100	38	96	75	77	0	38	24

of land with the maximum degree of belonging to the set the syntaxon represents; 2) all the typical phytosociological relevés, inevitably describe in a very abstract way the real vegetation.

Vegetation States, Vegetation stages, Vegetation types and Fuzzy Sets

Vegetation is a dynamic system whose states (relevés) are points in the ecological space (phase space). When fuzzy sets are defined on the basis of vegetation types, other fuzzy sets (environmental fuzzy sets, species fuzzy sets, life-growth form fuzzy sets, etc.) may be easily obtained by fuzzy relations or by matrix multiplication (Zimmerman 1984; Zhang 1988; Feoli and Zuccarello 1986, 1988, 1992). If such vegetation types are interpreted as vegetation stages of a dynamic process, then each of these fuzzy sets corresponds to a vegetation stage, which, by definition, is characterized by the object (species, life-growth form, environmental variable, etc.) that define the fuzzy set. In this context the fuzzy set defined by *Ostrya carpinifolia* represents the dynamical stage characterized by *Ostrya carpinifolia*, the fuzzy set defined by *Fagus sylvatica* represents the dynamical stage characterized by this species. In the same way, the fuzzy set defined by the "chamaephyte reptant" represents the dynamical stage characterized by this life-growth form. These fuzzy sets are vectors in a fuzzy system space with a dynamical meaning. The components of these vectors, i.e. the degree of belonging of all the considered elements (species, life-growth forms, vegetation types, etc.), may be used to infer the behaviour of each element in the different stages of the vegetation system under study.

Fuzzy inference

To inferring means to deduct facts by reasoning. Fuzzy sets are results of logical procedures codified by mathematical functions. They may be interpreted as mere data transformations and/or interpolations. All the matrices **F**(V;T), **F**(L;T), **F**(T;T), **F**(E;T) may be used to infer the possible vegetation states along the dynamical sequence characterized by single tree species. For example from matrix **F**(V;T) and Fig. 1a and 1b, we can infere where the vegetation types have their maximum identity, or response or performance, along the dynamical sequence. From the matrix **F**(L;T) we can infere the life-growth forms that are more likely to be present (more possible) in the different stages, while from matrix **F**(T;T) we can infer the behaviour of the trees passing through the different stages marked by themselves. From matrix **F**(E;T) (Table 7) we can infer the behaviour of environmental parameters along the dynamical sequence.

Is a fuzzy system space a naiveté?

Is the use of fuzzy set theory to study vegetation dynamics naive? Or more precisely, is the fuzzy system space proposed in this paper naive? One could say yes, because the mathematics is very simple: just the multiplication of matrices. Indeed fuzzy set theory is based on very simple mathematics, while vegetation dynamics is very complex and would require models of many non linear differential equations. But the parametrization and validation of such models would be impossible without permanent plots (Maarel & Werger 1978, Shugart, West & Emanuel 1981, Shugart 1984, Harrison & Shugart 1990). Actually the theory of succession was developed by using static data and a classification (Roberts & Morgan 1987) which define vegetation types with dynamic meaning (Mueller Dombois & Ellenberg 1974). This approach requires a considerable hypothetical element, present in the Braun- Blanquet approach. In this, several dynamical schemes of vegetation are layed out based on vegetation types as the stages or succesional sere (Weaver & Clements 1938). The answer to the question concerning the naiveté of the fuzzy system space proposed in this work is yes only, if we consider naive the Braun-Blanquet approach, that is, the approach based on classification. Certainly a scientific approach can not be judged more or less naive, only by the level of the mathematics used. The mathematics in our case is certainly simple, yet many inferences are possible by this simple mathematics, so many that it is truely surprising. Paraphrasing Beals (1973) we could say that the fuzzy system space may be considered mathematically naive but ecologically elegant.

Acknowledgements. This research has been supported by Italian CNR (International project CNR-CSIC "Nuove metodologie e modelli per lo studio dei processi di erosione e desertificazione negli ambienti mediterranei del sud-Europa") and Italian MURST (40 and 60%), we are grateful to them and to L. Orlóci, M. Dale, D. Roberts and J. Podani for linguistic revisions and comments.

References

Ashby, R.W. 1973. Some peculiarities of complex systems. Cybernetics Medicine, 9: 1-7.

Beals, E.W. 1973. Ordination: mathematical elegance and ecological naiveté. J. Ecol. 61: 23-35.

Bezdek, J.C. 1987. Some non-standard clustering algorithms. In: P. Legendre and L. Legendre (eds). Development in Numerical Ecology. pp. 225-287. Springer, New-York.

Bosserman, R.W. and R.K. Ragade 1982. Ecosystem analysis using fuzzy set theory. Ecological Modelling, 16: 191- 208.

Braun-Blanquet, J. 1964. Pflanzensoziologie. Grundzuge der Vegetationskunde. 3 Aufl. Springer, New York.

Cooley, W.W. and P.R. Lohnes, 1965. Multivariate data analysis. Wiley, New York.

Dale, M.B. 1988. Some Fuzzy Approaches to Phytosociology: Ideals and Instances. Folia Geobotanica et Phytotaxonomica 23: 239-274.

De Patta Pillar, V. AND L. Orloci, 1993. Taxonomy and perception in vegetation analysis. Coenoses 8: 53-66.

Feoli, E. 1977. On the resolving power of principal component analysis in plant community ordination. Vegetatio 33:119-136.

Feoli, E. 1984. Some aspects of classification and ordination of vegetation data in perspective. Studia Geobotanica 4: 7-21.

Feoli, E. and L. Orloci. 1979. Analysis of concentration and detection of underlying factors in structured tables. Vegetatio 40:49-54.

Feoli, E. and L. Orloci. 1991. The properties and interpretation of observations in vegetation study. In Feoli E. and L. Orloci (eds.). 1991. Computer Assisted Vegetation Analysis. Kluwer. Boston

Feoli, E. and M. Scimone.1985. Hierarchical diversity: an application to broad-leaved woods of Appennines. Giorn. Bot. Ital. 118:1-15.

Feoli, E. and V. Zuccarello. 1986. Ordination based on classification: yet another solution? Abstracta Botanica 10: 203-219.

Feoli, E. and V. Zuccarello. 1988. Syntaxonomy: a source of useful fuzzy sets for environmental analysis? Coenoses 3: 141-147.

Feoli, E. and V. Zuccarello. 1992. Fuzzy Sets and structural changes in forest succession: an example from broad leaved forests of N.E. Italy. Coenoses 7(2): 81- 90.

Feoli, E., P. Ganis AND L. Poldini. 1987. Relazioni tra corologia e descrizioni tassonomiche e morfologiche della vegetazione dei boschi ad *Ostrya carpinifolia* Scop. del Friuli venzia Giulia. Biogeographia 13: 155-178.

Flood, R.L. and E.R. Carson 1988. Dealing with Complexty. An Introduction to the Theory and Application of Systems Science. Plenum Press, New York.

Gittins, R. 1985. Canonical analysis: a review with applications in ecology. Springer, New York.

Goodall, D.W.1970. Studying the effects of environmental factors on ecosystems. In: Ecological Studies, Analysis and Synthesis. Vol. 1. Analysis of temperate Forest Ecosystems, ed. D.E. Reichle, Springer-Verlag, 19-29, Berlin.

Harrison E.A. and H.H. Shugart, 1990. Evaluating performance of an Appalachian oak forest dynamics model. Vegetatio 86: 1-13.

Klir and Folger 1988. Fuzzy sets, Uncertainy, and Information. Prentice Hall, Englewood Cliffs, New Jersey.

Landolt, E. 1977. Okologische Zeigerwerte zur Schweizer Flora. Ber. Geobot. Inst. ETH. 64: 64-207.

Maarel, E. van der and M. J. A. Werger, 1978. On the treatment of succession data. Phytocoenosis 7: 257-278.

Marsili-Libelli, S. 1989. Fuzzy clustering of ecological data. Coenoses 4: 95-106.

Mueller-Dombois, D. and H. Ellenberg, 1974. Aims and methods of vegetation ecology. J. Wiley, New York.

Nijkamp, P., Leitner H. and N. Wrigley 1985. Measuring the Unmeasurable. NATO ASI Series D: #22. Martinus Nijhoff Publishers, Dordrecht.

Orlóci, L 1991a. Poorean approximation and Fisherian inference in bioenvironmental analysis. Advances in Ecology, 1: 65-71.

Orlóci, L 1991b. Statistics in ecosystem survey: computer support for process-based sample stability test and entropy/information inference. In Feoli E. and L. Orloci (eds.). 1991. Computer Assisted Vegetation Analysis. Kluwer. Boston

Poldini, L. 1982. *Ostrya carpinifolia*-rich woods and bushes of Friuli-Venezia Giulia. Studia Geobotanica 2: 69-122.

Roberts, D.W. 1986. Ordination on the basis of fuzzy set theory. Vegetatio 66: 123-143.

Roberts, D.W. 1987. A dynamical system perspective on vegetation theory. Vegetatio 69: 27-33.

Roberts, D.W. 1989a. Analysis of Forest Succession with fuzzy graph Theory. Ecological Modelling 45: 261-274.

Roberts, D.W. 1989b. Fuzzy systems vegetation theory. Vegetatio 83: 71-80.

Roberts, D.W. and P. Morgan, 1987. Classification and models of succession. In D. Ferguson et al. (eds). Land Classification based on Vegetation: Applications for resource management. U.S.D.A. For Serv. Internat. Res. St. Gen. Tech. Rep. INT-257. Ogden, UT. 49-53.

Rosen 1979. Fundamentals of Measuraments and Representation of Natural Systems. North-Holland, New York.

Shugart, H.H. 1984. A theory of Forest Dynamics. The ecological Implications of Forest Succession Models. Springer-Verlag, New York.

Shugart, H.H., D.C. West and W.R. Emanuel, 1981. Pattern and Dynamics of Forests: An Application of simulation models. In West D.C., H.H. Shugart and D.B. Botkin (eds). Forest Succession. Concept and Application. Springer- Verlag, New York.

Weaver, J.E. and F.E. Clements, 1938. Plant Ecology. Mc Graw-Hill, New York.

Westhoff, V. and E. Maarel van der, 1978. The Braun- Blanquet approach. 2nd ed. In R.H. Whittaker (ed.), Classification of Plant Community. Junk, The Hague.

Yager, R.R. 1979. On the measure of fuzziness and negation. Part I: Membership in the unit interval. Int. J. Gen. Syst. 5: 221-229.

Zadeh, L.A. 1965. Fuzzy sets. Inform. Control, 8: 338- 353.

Zadeh, L.A. 1973. Outline of a new approach to the analysis of complex systems and decision processes. IEEE Trans. Syst. Man Cybern., 2: 28-44.

Zhang, D. 1988. An index to measure the strength of relationship between community and site. Ecological Modelling, 40: 145-153.

Zimmerman, H.J. 1985. Fuzzy set Theory and its Applications. Kluwer-Nijhoff Publishing, Boston, Dordrecht.

Manuscript received: October 1993